Emerson (W.) K

CALCULATION, LIBRATION, AND MENSURATION;

Or the ARTS of

RECKONING, WEIGHING, AND MEASURING.

BEING A

MECHANICAL WORK,

ADAPTED

To the Buſineſs and Practice of TRADESMEN and ARTIFICERS, in the ſhorteſt Method poſſible; and deſigned purely for common Uſe.

Pondere, menſura, et numero, Deus omnia fecit.

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THE PREFACE.

I Believe there never was a courſe of the Mathematicks printed, but what included the practical parts thereof, and ſuch things as are of conſtant and daily uſe among mankind. Therefore, leſt ours ſhould appear defective in this point, we ſhall comply with this cuſtom; and to oblige the lower ſort of readers (for every one is not deſtined to ſtudy the ſublime parts of the Mathematicks) I have compoſed the following Treatiſe.

In the firſt part I have put together a great number of rules for the reckoning up of ſuch things as commonly occur in trade and buſineſs; wherein I have endeavoured to ſuit the capacities of the loweſt claſs of tradeſmen and artificers, by laying down ſuch ſhort rules, for computing the values of things, as may with eaſe be retained in the memory, are readily and eaſily wrought, or rather performed in the mind, which are to be uſed as occaſion offers; and therefore muſt be very ſerviceable to ſuch as never had an opportunity of learning Arithmetic. And even to ſuch people as underſtand numbers, theſe rules will ſave them ſo much trouble and time. For men cannot always allow time to ſit down and compute every trifling matter, which may be done at once by this way of reckoning. For few of theſe rules need the uſe of Arithmetic; and thoſe that do require but an eaſy calculation. But where there is any danger of making miſtakes, it is proper, for thoſe that underſtand it, to uſe a little Arithmetic. And that no doubt or obſcurity may ap-

pear in theſe rules, I have given examples thereof, in the common caſes of trade; ſo that the meaneſt reader cannot be at a loſs for underſtanding them. And before this part, the common definitions are prefixt, alſo an account of weights and meaſures, tables of ſpecific gravity, the practice of workmen in meaſuring their work, and ſuch other things as are uſeful in a Treatiſe of this kind.

In the ſecond part are delivered ſuch common problems, as are requiſite in the art of Meaſuring, being ſuch as are introductory to what is to follow. Here we treat of the meaſuring of lines; and how, by having one line in a figure given, to find another. To this part is prefixt the deſcription and uſe of the ſliding rule, an inſtrument of great diſpatch in the art of meaſuring.

The third part teaches how to meaſure the areas of plane figures, and the ſurfaces of ſolids, and this both arithmetically with the pen, and inſtrumentally with the ſliding rule. In this part it is alſo ſhewn, how theſe fixt numbers are found, which are made uſe of in the different methods of finding the area of the circle, by the ſliding rule.

In part the fourth, we have the menſuration of ſolids, eſpecially the moſt common; and particularly the meaſuring of timber; and this both by Arithmetic with the pen, and alſo by the ſliding rule, when not too troubleſome. In this part it is alſo ſhewn how all the fixt numbers are found, which are made uſe of on the ſliding rule, for meaſuring cylinders and ſpheres.

The fifth and laſt part is gauging, contained in a narrow compaſs. Here it is ſhewn how to find the areas of plain figures, and the contents of ſolids, according to the common method of gauging; that is in ale or wine gallons, inſtead of inches or feet. How to inch any tun; and a general rule for gauging all caſks; and how to find the ullage; alſo how to gauge malt. To this part is prefixt a ſhort account of the *ſliding rule adapted to this buſineſs; an inſtrument which very much ſhortens all calculations in gauging. And in this part is ſhewn likewiſe, how to find the fixt numbers and gauge points, made uſe of on the rule, and how to work with them. In all theſe articles are given proper examples, to make the rules plain, which are wrought in the ſhorteſt method, both by the pen, and alſo with the ſliding rule, when it can conveniently be done.*

I have given no demonſtrations here, of any of theſe rules; for the practical Mechanic does not want them, and a mathematical reader may have them elſewhere, as in the Geometry, Conic Sections, Algebra, &c.

There is no ſtate of life which does not require the uſe of ſome ſort of meaſures, either in length or capacity, dry or liquid; on theſe moſt trades depend, whether it be thoſe that procure food for us, or thoſe that furniſh us cloathing. Menſuration is the ground of all arts, and of all things depending on trade, without which they could not ſubſiſt.

In meaſuring and gauging, the practitioner muſt underſtand common Arithmetic, without which theſe rules cannot be put in practice; and alſo decimals, as all ſorts of computations are eaſieſt made thereby; and particularly contractions in multiplication and diviſion, which often ſave long calculations. Tho' in ſuch caſes, I would adviſe to make uſe of logarithms, which in moſt caſes will be ſhorter.

As geometrical rules are made uſe of for ſuch purpoſes as require calculations to be made to the utmoſt exactneſs; ſo the ſhort rules in this Treatiſe, are moſt of them only approximations, being contrived to compute, or meaſure things for common uſe with expedition, and which require not that degree of exactneſs. They are adapted for preſent extemporary buſineſs, and for ſuch things as require to be done off hand.

When any plane figure or ſolid body is propoſed to be meaſured, we have no rule to determine it to be any particular geometrical figure or ſolid, but by gueſs. And therefore it is to be looked on as a mechanical figure or ſolid; and muſt be meaſured by ſome ſhort mechanical rule, ſuch as ſuits it, and its ſhortneſs will make amends for other defects. For by ſuch a rule the content may be found as truly, as the dimenſions can be taken, and that is ſufficient. And all ſuch rules muſt be eſteemed good rules; when they come very near, at a medium; as if in one caſe they give a ſmall matter too much, and in another caſe a ſmall matter ſhort of the truth, we may admit them as true rules; ſince the figure we meaſure may not have that geometrical exactneſs that we ſuppoſe; and it is mere chance if it has.

W. Emerſon.

Marks and Abbreviations.

+ to be added.

— to be ſubſtracted.

× to be multiplied by.

÷ to be divided by.

√ the ſquare root of.

a^2 the ſquare of a.

a^3 the cube of a, &c.

= is equal to.

$a : b :: c : d$, as a to b, ſo is c to d.

AG, ale gallons.

WG, wine gallons.

MB, malt buſhels.

GP, gauge point.

Calculation, Libration, &c.

DEFINITIONS.

1. *COMPUTATION* or Numbering is the Art of Reckoning; which is ſometimes performed barely by the memory; but generally by the help of theſe figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

2. *Libration* is the Art of Weighing any ſort of bodies. This is performed by ſome proper inſtrument, as a pair of ſcales, a ſteelyard, &c. and by the help of certain fixt weights, agreed on to be the ſtandard, for weighing all other things by.

3. *Menſuration* is a mechanical art, teaching how to find the content of any ſurface, or ſolid body, from the dimenſions thereof, taken by actually meaſuring them. The content of any ſurface is commonly called the *Area* of it.

4. *A Line* is the length or diſtance between any two points; and this length is eſtimated by the number it contains, of ſome known meaſure, as yards, feet, inches, &c.

5. *A Superficies* is that which hath length and breadth, and its quantity is meaſured by the number of ſquares it contains, whoſe ſides are linear units, of that meaſure it is meaſured by.

6. *A Solid* is that which hath three dimenſions, length, breadth, and thickneſs. And its content is eſtimated by the number of cubes it contains, whoſe ſides are linear units of that ſort of meaſure it is meaſured by.

7. *A right Angle* is that made by one line DC 14.
ſtanding upright upon another BA, ſo that CD

Fig. lean no more to one ſide than to another; thus CDA is a right angle, and alſo CDB. If CD is not upright, it is an *oblique angle*; an *acute angle* is leſs, and an *obtuſe* one greater, than a right angle.

1. 8. *Parallel lines* are thoſe that are every where equidiſtant, as AB and CD.

14. 9. *A Perpendicular*, is a line which ſtands at right
15. angles upon a line or a plane, and inclines to no ſide. This is the neareſt diſtance from any point to a line or a plane, as CD.

14. 10. *A Triangle* is a figure contained under three
15. right lines. If one angle be right, it is a *right angled triangle.*

16. 11. *A Square* is a figure contained by 4 equal ſtreight lines and 4 right angles, as ABCD.

17. 12. *A Parallelogram* is a figure contained by
18. four right lines, ſo that the oppoſite be equal and parallel.

18. 13. *A Trapezoid* is a four ſided figure, whereof ſome two oppoſite ones are parallel, as ABCD.

19. 14. *A Trapezium* is any figure included by four lines, as ABCD. A line drawn thro' the oppoſite angles is called a *Diagonal*, as AC.

21. 15. *A Polygon* is a figure conſiſting of any num-
22. ber of ſides. If all the ſides and angles are equal, it is a *regular Polygon*; if it have three ſides, it is called an *equilateral Triangle*; if four, a *Square*; if five, a *Pentagon*; if ſix, a *Hexagon*, &c.

23. 16. *A Circle* is a figure bounded by the curve line ABCD, called the Circumference, in which all lines drawn from the *Center* C to the circumference are equal. Any line AD paſſing thro' the center C is called a *Diameter*; and any line CD, drawn from the center C to the circumference, is called the *Radius*. Any right line drawn through it is called a *Chord*.

17.

17. *A Cube* is a ſolid contained under 6 equal ſquares ſtanding at right angles to one another; like a die. Fig. 38.

18. *A Parallelepipedon* is a ſolid contained under 6 planes, whereof every two oppoſite ones are parallel. 40.

19. *A Priſm* is a ſolid whoſe two ends are equal and like, and parallel to one another. 41, 42.

20. *A Cylinder* is a ſolid whoſe ends are two equal circles, parallel to one another. 39.

21. *A Pyramid* is a ſolid whoſe baſe is any plane figure, and the ſides all triangles meeting in a point called the Vertex, as D. 44.

22. *A Cone* is a ſolid ABC contained between a circular baſe, and a convex ſurface ending in a point A called the Vertex. 43.

23. *A Poliedron* is a ſolid contained under ſeveral planes, if all the planes be any regular poligon, it is called a *regular body*, of which there are but five. 1. The *Tetraedron*, has four faces equilateral triangles. 2. The *Cube* has ſix ſquares. 3. The *Octaedron*, 8 triangles. 4. The *Dodecaedron*, 12 pentagons. 5. The *Icoſaedron*, 20 triangles.

24. *A Sphere* ABCD, is a ſolid contained under one curve ſurface; within which there is a point called the *Center*, from which all lines drawn to the ſurface are equal, and called the *Radius*. Any line paſſing thro' the center is a *diameter*. 33.

25. *A Segment* is any part of a figure or ſolid cut off by a line or a plane.

26. *A Fruſtum* is the thick end of a ſolid cut off by a plane parallel to the baſe.

27. *Vaults* are curved arches, that ſpring from the oppoſite ſide walls; and meet in a right line over the middle of the building. If they meet in an angle, they are called Gothic arches. The arches of a bridge are in this form.

28. *Dome's* or *Cupolo's* are arched roofs that meet in a point at the top; the baſe being a circle or poligon.

29. *A Saloon* is a flat roof or cieling, joined to the ſide walls by arches.

30. *A Groin* is when a vaulted roof is interſected by ſome other vaults.

31. *A Meaſure* is a certain length made uſe of to meaſure another greater length by, in order to know how many of theſe the greater contains. When a length is to be meaſured, it is meaſured by a line, and is called *lineal meaſure.* When a ſurface is to be meaſured, it is done by the ſquare of that line, and is called *ſuperficial meaſure.* If a ſolid is to be meaſured, the meaſure is the cube of that lineal meaſure, and then it is called *ſolid meaſure.* Thus a foot in length meaſures any other length; and a ſquare foot, any ſurface; and a cube foot any ſolid; and ſhews how many feet each contains. And the like for inches, yards, &c.

In meaſuring any ſurface or ſolid, all the dimenſions muſt be taken in one and the ſame kind of meaſure, except it be otherwiſe expreſſed.

A Table of Numbers.

12 Units	make	a Dozen,
12 Dozen		a Groſs,
20 Units		a Score,
5 Score		a ſhort Hundred (Є)
6 Score		a long Hundred (C)
10 Hundred		a Thouſand (M)

A Table of Money.

4 Farthings	make	1 Penny (d)
4 Pence		1 Groat,
12 Pence		1 Shilling (s)
5 Shillings		1 Crown,
6 Shil. 8 d.		1 Noble,
10 Shillings		1 Angel,
13 Shill. 4 d.		1 Mark,
20 Shillings		1 Pound (l)
21 Shillings		1 Guinea.

A Table of Time.

60 Seconds (″)	make	1 Minute (′)
60 Minutes		1 Hour,
24 Hours		1 Day natural,
7 Days		1 Week,
4 Weeks		1 Month,
365¼ Days		1 Year.

Goods ſold by the Dozen are,

Buttons, candles, hats, ſtockings, lint, bottles, bruſhes, China ware, pigeons, apricots, and peaches.

Goods ſold by the Groſs are,

Buttons, beads, tobacco-pipes, combs, comb-caſes, playing cards, knives, thread, ſilk hair, tobacco-boxes, buckles, ink-horns, bodkins, pepper-boxes, tinder-boxes, waſh-balls, and ſpectacle-caſes.

Goods ſold by the Score are,

Paper, walking-ſticks, marbles.

Goods ſold by the ſhort Hundred are,

Capers, thrums, ginger, cloves, indigo, pack-thread, quick-ſilver, cotton-wool, braſs and tin-work, ſeveral ſorts of ſkins, braſs and tin implements.

Goods ſold by the long Hundred are,

Herring, ling, cod, and all ſtock-fiſh, balks, clap-boards, nails, fiſh-hooks, pipe-ſtaves, bow-ſtaves, deals, ſpurs, outlandiſh cloth, lamb-ſkins, ſeveral ſorts of outlandiſh cloth, and eggs.

Goods ſold by the Thouſand are,

Paſte-boards, tennis-balls, lemons, oranges, boards, quills, teazles, tiles, bricks, tacks, tenter-hooks, thimbles, pins and needles, billets, lanthorn-horns, barrel-hoops, ox-horns, ſquirrel-ſkins, ſlates, &c.

Standard gold is 11^{oz} fine, and 1^{oz} alloy.

Standard ſilver is $11^{oz}.2^{dw}$ fine, and 18^{dw} alloy.

A pennyweight of gold coin is worth 3 s. 10d$\frac{1}{2}$, which is near 2 d. a grain.

44$\frac{1}{2}$ Guineas make 1 pound Troy; and 1 guinea, 129 grains.

An ounce of ſilver with alloy is worth 5 ſhillings.

An ounce of gold is worth 3 l. 18 s.

22 Pence in copper is about 1 l. Averdupoiſe.

A Table of Averdupoiſe weight.

16 Drams	make	1 Ounce (oz)
16 Ounces		1 Pound (lb)
14 Pounds		1 Stone (St)
8 Stone		1 Hundred weight (C)
20 Hundred		1 Tun. (T)

8 lb make a ſtone among the London butchers.

A Table of Troy Weight.

20 Mites	make	1 grain, (gr)
24 Grains		1 Penny weight, (dw)
32 gr'. wheat		1 Penny weight,
20 Penny weight		1 Ounce, (oz)
12 Ounces		1 Pound. (lb)

The Apothecaries divide the Troy ounce thus, 20 grains make a ſcruple (℈), 3 ſcruples a dram (ʒ), 8 drams an ounce (℥).

By Averdupoiſe weight, is weighed mercery ware, and groceries, and all things that can waſte, as pitch, tar, roſin, wax, tallow, flax, hemp, fleſh, butter, cheeſe, ſalt, candles, ſoap, ſtarch, ſpices, coffee, tea, chocolate, ſilk, bread in the country, and ſometimes oil; alſo copper, tin, iron, ſteal, lead, &c.

By Troy weight is weighed gold, ſilver, jewels, amber, apothecaries drugs, bread, liquors, precious ſtones.

Weights compared.

The Averdupoiſe pound is 7000 grains, the Troy pound 5760 grains.

The Averdupoiſe ounce is 437½ grains, the Troy ounce 480 grains.

The Troy pound is to the Averdupoiſe pound, as 9 to 11, or rather, as 14 to 17.

The Averdupoiſe ounce is to the Troy ounce, as 10 to 11, or more exactly, as 73 to 80.

The London pound is to the *Paris* pound, as 100 to 109.

The Troy ounce is to the *Paris* ounce, as 64 to 63.

And the Averdupoiſe ounce is to the *Paris* ounce, as 63 to 68.

A Table of long Meaſures.

12 Lines	make	1 Inch,
3 Barley-corns		1 Inch,
12 Inches		1 Foot,
3 Feet		1 Yard,
5 Feet		1 Pace,
2 Yards		1 Fathom,
$5\frac{1}{2}$ Yards		1 Pole or Rod,
4 Poles, 100 links		1 Chain,
40 Poles		1 Furlong,
8 Furlongs		1 Mile,
$69\frac{1}{2}$ Miles		1 Degree on the earth.

Alſo 1 finger or thumb is an inch; 4 inches a hand; 9 inches a ſpan; 2 ſpans a cubit; 5 ſpans an ell; 16 nails a yard; 7 yards 1 rod in the north; a geographical mile 2030 yards; the *Engliſh*, *French* and *Swediſh* feet are to one another, as 1 to 1.066, and 0.97; a toiſe is 6 *French* feet.

A Table of ſquare Meaſures.

144 Inches	make	1 Foot,
9 Feet		1 Yard,
$30\frac{1}{4}$ Yards		1 Pole or Perch,
100 Feet		1 Square,
16 Poles		1 Chain,
40 Poles		1 Rood,
10 Chains		1 Acre,
4 Roods		1 Acre,
640 Acres		1 Mile.

A Table of ſolid Meaſure.

1728 Inches	make	1 Foot,
27 Feet		1 Yard.

A Table of dry Meaſure.

268.8 cub. Inches, or 4 Quarts,	make	1 Gallon,
2 Gallons		1 Peck,
4 Pecks, or 2150.4 cub. Inches		1 Buſhel,
8 Buſhels		1 Quarter,
4 Quarters		1 Cauldron,
10 Quarters		1 Laſt.

Alſo 2 pints make a quart; 2 pottles 1 gallon; 2 buſhels a ſtrike; 4 buſhels a comb; 2 weys a laſt.

Water meaſure is upheap'd meaſure, or elſe allows 5 ſtriped pecks to the buſhel.

By this, grain, coals, ſalt, lime, are meaſured.

A Table of liquid Meaſure.

4 Jacks	make	1 Pint,
2 Pints		1 Quart,
4 Pints		1 Gallon,
231 cub. Inches		1 Gallon of wine,
282 cub. Inches		1 Gallon of ale or beer,
$8\frac{1}{2}$ Gallons		1 Firkin or Anchor,
6 Firkins, or 51 Gallons		1 Hogſh. of ale or beer,
63 Gallons		1 Hogſhead of wine,
4 Hogſheads		1 Tun.

Alſo 2 gills make a pint; 2 quarts, a pottle; 18 gallons, a runlet; 42 gallons, a terce; 2 terces, a puncheon; 2 firkins, a kilderkin; 4 firkins, a barrel; 2 hogſheads, a pipe or butt.

Note, 8 gallons make a firkin of ale, and 9 of beer, in London; and the hogſhead proportionable. Alſo 4 gills make a pint there.

A *Paris* pint contains 49 cubic inches.

Oil, honey, &c. are meaſured by wine meaſure, and ſometimes by weight Averd. Alſo brandy, ſpirits, mead, perry, cyder, vinegar, verjuice, ſweets, mum, &c.

A Wincheſter buſhel ought to be 8 inches deep, and $18\frac{1}{2}$ inches diameter.

A ſtraked buſhel is to the ſame unheaped as 3 to 4; therefore the heap is $\frac{1}{3}$ the buſhel; or, as ſome ſay, $\frac{1}{4}$ thereof.

The old wine gallon was 224 cubic inches.

The old corn gallon 272 cubic inches.

The corn gallon is to the ale gallon, as 20 to 21.

A hogſhead of water (wine meaſure) is about $8\frac{1}{3}$ ſolid feet; the ſame by ale meaſure at 51 gallons per hogſhead. And wine meaſure to ale meaſure, as 9 to 11.

A cubic foot is to the Wincheſter buſhel, as 4 to 5; or exactly, as 45 to 56.

The Engliſh foot is to the *Paris* foot, as 107 to 114.

SCHOLIUM.

However theſe things may have varied; yet it is very probable that, originally, a hogſhead of wine and ale was the ſame in quantity; and a tun of water juſt a tun weight.

Hence we may ſuppoſe, that anciently 8 cubic feet of water made juſt a hogſhead, and 4 hogſheads a tun. And conſequently that a tun weighed 2000 lb.

Likewiſe that a hundred weight was then but 100 lb; for this multiplied by 20, makes 2000 lb. alſo for a tun; and was increaſed afterwards by merchandizing, upon account of the waſte in goods.

Alſo if a buſhel of good wheat weighs $62\frac{1}{2}$ lb, then the quarter would be 500 lb, and the chaulder 2000 lb. which is alſo a ton. So that originally, what was a ton in meaſure was alſo a ton in weight. A ton in weight has been the common ſtandard of all things.

Weight and Meaſure compared.

1 lb. Troy of water is $22\frac{3}{4}$ inches ſolid.

1 lb. Averdupoiſe of water is $27\frac{1}{2}$ ſolid inches.

1 inch of water weighs .58 ounces Averdupoiſe.

A cubic foot of water weighs 76 lb. Troy, or $62\frac{1}{2}$ lb. Averdupoiſe, which is 1000 ounces, and contains $6\frac{1}{8}$ ale gallons.

A wine gallon of water weighs 8.35 lb. Averdupoiſe.

An ale gallon of water weighs 10.2 lb. Averdupoiſe.

A ton of water weighs $18\frac{3}{4}$ hundred weight.

A cylinder of water of d inches diameter, and a yard high, contains $\frac{1}{10}dd$ ale gallons; and weighs $1\frac{1}{5}dd$ pounds Averdupoiſe.

A Table of ſpecific Gravities, ſhewing the weight of a cubic Inch, of ſeveral bodies, in ounces Averdupoiſe.

	oz.		oz.
Fine gold	11.366	Caſt iron	4.090
Standard gold	10 930	Marble	1.569
Quickſilver	8.102	Common glaſs	1.493
Lead	6.554	Flint	1 490
Fine ſilver	6.418	Portland ſtone	1.488
Standard ſilver	6.096	Free ſtone	1.465
Copper	5.208	Common ſtone	1.446
Plate braſs	4.832	Brick	1.157
Caſt braſs	4.630	Alabaſter	1.084
Steel	4.544	Ivory	1.055
Iron	4.423	Horn	1.055
Pewter	4.323	Water	0.578
Tin	4.236		

A Table ſhewing the weight of a cubic foot of ſeveral ſorts of Bodies, Averdupoiſe weight.

	lb.	oz.		lb.	oz.
Wheat —	48	0	Lignum vitæ	82	15
Oats —	29	8	Butter —	58	12
White peaſe	50	8	Pitch —	71	14
Barley —	41	2	Mahogany	66	7
Malt —	30	4	Box —	64	6
Field beans	50	8	Milk, Urine, and Sea water	63	2
Wheat meal unſifted —	31	0			
Rye meal unſifted	28	4	Rain water	62	8
Bay ſalt —	54	1	Red wine	61	14
White ſea ſalt	43	12	Bees wax	59	11
Sand —	85	4	Linſeed oil	58	4
Newcaſtle coal	67	12	Brandy	57	15
			Oak —	57	8
Pit coal (Wedneſbury) —	63	0	Oil olive	57	1
Gravel	109	5	Beech	53	6
Wood aſhes	58	5	Elm, Aſh —	49	1
Pump water	62	8			
Chalk —	112	10	Cedar —	38	7
Allum —	107	2	Fir —	36	4
Clay —	107	0	Cork —	14	8
Honey —	90	10	Air —	0	$1\frac{1}{4}$

REMARKS.

20 cubic foot of hay is about a hundred weight.

56 lb. of butter make a firkin, which is 4 ſtone, Averdupoiſe.

When wheat is at 5 ſhillings the buſhel, the penny white loaf is to weigh $7\frac{1}{4}$ oz. out of London.

If a miller carries and recarries his corn, he has a quarter of a peck per buſhel, for toll and waſte. But if the owner carry it, he has but half ſo much.

40 feet of unhewn timber, or 50 feet of hewn timber, is reckoned a load or ton.

60 ſolid feet of Newcaſtle coals is a chaldron, or 36 buſhels.

A buſhel of wheat weighs about 60 lb. Averdupoiſe, which is 4 ſtone 4 lb; but ſome weighs more.

A load of hay is 36 truſſes, and a truſs four ſtone, or half a hundred.

The ſeveral ways of meaſuring different ſorts of Work.

Different work is computed by different meaſures. In meaſuring boards, timber, glaſs, ſtone; the dimenſions are taken in feet and inches; and the content caſt up in feet.

In caſks, the dimenſions are taken in inches, and the content caſt up in ſolid inches, or elſe in gallons.

In painting, plaiſtering, paving, flooring, wainſcot; the dimenſions are taken in feet or yards; and the content caſt up in yards.

In roofing, tyling, ſlating, raftering, partitioning, flooring; the dimenſions are taken in feet, and the content given in ſquares of 100 ſquare feet.

Walling is meaſured in feet or yards, and the content given in rods, containing 272 ſquare feet; or, in the North country, the rod contains 7 ſquare yards.

Land is meaſured by Gunter's chain, and caſt up in acres, at 10 chains an acre.

Earth works, as cellars, vaults, &c. are meaſured by the yard ſolid.

In any rule, &c. to meaſure with, it is the beſt way to have it divided into decimal parts; for the content of any thing is ſooneſt caſt up, when the dimenſions are taken in decimals. And the ſhorteſt

eſt way is to work by logarithms; adding the logarithms, inſtead of multiplying the numbers; and ſubtracting inſtead of dividing.

The Practice of Workmen.

Glaziers take dimenſions to a quarter of an inch, and never make any allowance for round or oval windows; but meaſure the circumſcribing parallelograms.

In plaiſtering, deductions muſt be made for doors and windows; but ſome make no deductions.

Painters, in taking the height, gird the ſtring into all the hollows, and round all the ſwellings of the pannels, &c. And in taking the height of a door, or the breadth, they add the thickneſs. Their price is according to the number of times the colour is laid on. Deductions are made for chimnies, caſements, &c.

Joiners, in meaſuring the height of a room, put the ſtring into all the ſwellings and mouldings. But in taking the length, they meaſure in a ſtrait line. Deductions are made for all vacancies, as windows, chimneys, &c. In partitioning, deductions muſt be made for doors and windows.

In flooring, the hearth-ſtone is deducted, except there be a border round it. Window ſhutters and doors are commonly reckoned meaſure and half; becauſe they are worked on both ſides.

In roofing, tiling, ſlating, it is cuſtomary to reckon the flat and half of building within the walls, for the meaſure of the roof, when it is a true pitch; that is, when the rafters are $\frac{3}{4}$ the breadth of the building. For workmanſhip alone, ſky-lights, chimney ſhafts, &c. are deducted. But finding materials and work, takes in all, except places of 9 or 10 feet area. Some never deduct for window ſhafts and ſky-lights at all; they are ſo troubleſome to the workmen.

In tiling and ſlating, ſky-lights and chimney ſhafts are deducted. The angle of the roof which bends inwards is called a *Valley*, and the angle bending outwards is called a *Hip*.

In walling a houſe, deductions muſt be made for doors and windows.

In meaſuring chimnies, gird round the middle for the breadth, and the height of the ſtory is the length.

In meaſuring timber, it is cuſtomary for the buyer to take the girt any where he pleaſes, between the greater end and the middle of the tree. Branches of two feet compaſs are reckoned timber; any thing leſs is not timber. When the bark is on, deduct $\frac{1}{10}$ or $\frac{1}{12}$ the circumference.

PART I.

Computing and Weighing.

PROB. I.

To reduce Guineas into Pounds.

1 RULE.

TAKE the tenth part of the Guineas, and add the half thereof to the guineas, for the pounds.

What remains is ſhillings.

Examp.

In 674 Guineas, how many Pounds?

The tenth of 674 (dropping 4) is 67, half of 67 (dropping 1) is 33. Then 33 added to 674, gives 707 pounds, to which put the 14 ſhillings remaining.

2 RULE.

Call the guineas ſhillings, which reduce into pounds, and add to the guineas.

Example.

How many Pounds in 168 Guineas?

Here 168 ſhillings is 8l. 8s. which added to 168, gives 176l. 8 s.

Cor. *To reduce pounds to guineas, ſubtract the 21ſt part thereof.*

PROB. II.

Having the price of 1, *to know what* 100 *comes to?*

1 RULE.

For every farthing in the price of one, reckon twice as many ſhillings and once as many pence.

Examp.

If 1 *coſts* 9½ *pence, what will* 100 *come to?*

9 pence halfpenny is 38 farthings.		
Then twice 38 is	76 s.	0 d.
And 38 pence	3	2
	79 s.	2 d. = 3 l. 19 s. 2 d.

2 RULE.

For every ſhilling of the price reckon 5 pounds.
And for every 3d. beſides reckon 25 ſhillings.
And for a penny 8 s. 4 d.

Examp.

What comes 100 *to, at* 7 s. 10 d. *a piece?*

The ſhillings	—	35	0	0
The 3 pences	—	3	15	0
A penny	—		8	4
		39	3	4

PROB. III.

Having the price of 100 *given, to find the price of* 1.

RULE.

Take half as many farthings as there are ſhillings in the price; abating the 50th part of the whole number of farthings.

Examp.

If 100 *coſt* 33 l. *what is that a piece?*

Whole num. farthings	660	
Half the number	330	
$\frac{1}{50}$ the whole, ſub.	13	
	317	farthings.
	$79\frac{1}{4}$	pence.
Anſ.	6 s. 7 d$\frac{1}{4}$.	

PROB. IV.

Having the price of 1, *to know what the long hundred will coſt.*

1 RULE.

How many pence you give for each one thing,
So many angels the long hundreds bring.

Examp.

If one thing coſt $3\frac{1}{4}$ d. *what will* 120 *be?*

$3\frac{1}{4}$ angels $32\frac{1}{2}$ s. = 1 l. 12 s. 6 d.

2 RULE.

2 RULE.

Take half as many pounds as 1 coſts pence; adding 2 s. 6 d. for every odd farthing.

Ex.

If 1 *coſts* $13\frac{3}{4}$ d. *what is the charge of* 120?

Half of 13 is $6\frac{1}{2}$	6 l.	10 s.	0 d.
3 half crowns for $\frac{3}{4}$		7	6
Anſ.	6	17	6

PROB. V.

By knowing the price of the long hundred, to know what 1 *coſts.*

1 RULE.

For every pound reckon 2 pence, and for every half crown more a farthing.

Ex.

If the long hundred (120) *coſts* 7 l. 15 s. *what does* 1 *coſt?*

7 two pences	1 s.	2 d.
6 farthings		$1\frac{1}{2}$
Anſ.	1	$3\frac{1}{2}$

2 RULE.

Double the price, then the number of pounds call pence, and the number of crowns over, call farthings.

Ex.

If deals be 18 l. 16 s. 3 d. *for the long hundred, what does* 1 *coſt?*

double, 37 l. 12 6 = 3 s.—1½ d. Anſ.

PROB. VI.

Having the price of 1, *to know what the thouſand comes to.*

1 RULE.

For every 6 pence of the price, reckon 25 pounds; and for every penny more, 4 l. 3 s. 4 d. or 4⅙ pounds.

Or ten times the price of 100, is the price of a thouſand.

Ex.

Suppoſe 1 *coſts* 13 s. 4 d. *what does* 1000 *coſt?*

26 ſix pences	650		
4 times 4 l. 3 s. 4 d.	16	13	4
Anſ.	666	13	4

2 RULE.

Take as many pounds as there are farthings in the whole price. Then add as many pounds as there are ſix pences in the price. Alſo for every 1½ d. above, add 5 ſhillings, and 10 d. for each farthing.

Examp.

If 1 *coſts* 14¾ *pence, what will* 1000 *coſt?*

59 farthings	59 l.	s.	d.
2 ſix pences	2		
1½ d.	0	5	0
5 farthings		4	2
Anſ.	61	9	2

PROB.

PROB. VII.

Having the price of a thouſand, to find the price of one.

RULE.

Take as many farthings as pounds in the price, abating the 25th part thereof.

Ex.

At 564 l. 18 s. 9 d. *per thouſand, what does* 1 *coſt?*

565 farthings	565
$2\frac{1}{25}$ part ſub.	22
Anſ.	543 farthings = 11 s. $3\frac{3}{4}$ d.

PROB. VIII.

Having the price of 1, *to find that of a dozen.*

RULE.

For every penny in the price reckon a ſhilling, and 3 pence for each odd farthing.

Ex.

If 1 *coſts* $16\frac{1}{2}$ *pence, what will* 12 *be?*

Anſ. 16 s. 6 d.

Cor. *The price of a dozen being given; the price of* 1 *is found, by reckoning ſo many pence as there are ſhillings; or by taking* $\frac{1}{12}$ *the price.*

PROB.

PROB. IX.

Having the price of 1 thing; to find the price of a groſs.

RULE.

Find the price in pence, and take $\frac{6}{10}$ of it, for the pounds.

Ex.

If one coſts 3 s. $11\frac{1}{2}$d. *what is a groſs?*

3 s. $11\frac{1}{2}$ $47\frac{1}{2}$ pence
.6

Anſ.   28.50 or 28 l. 10 s.

PROB. X.

Having the price of a groſs, to find the price of one.

RULE.

Take the third part of the price in ſhillings, for the price of 1 in farthings.

Ex.

At 7 l. 9 s. 6 d. *the groſs, what is the price of* 1?

7 l. 9 s. 6 d. is $149\frac{1}{2}$ ſhillings, or 150 s.
3)150(50 far. = $12\frac{1}{2}$ d.   Anſ.

PROB. XI.

Having the price of a gallon of liquors; to find the price of a ton.

RULE.

Put the price into pence, and call them ſo many guineas; adding quarter guineas for farthings.

Ex

Ex.

If brandy be ſold for 6 s. 10¾ d. *a gallon, what is that a ton?*

6 s. 10¾ d. is	82¾ d.		
82 guin.	82	0	0
	4	2	0
¾ d.		15	9
Anſ.	86	17	9

Cor. *The price of a hogſhead is had, by taking a quarter of this ſum.*

PROB. XII.

Having the price of a ton of liquors; to find that of a gallon.

RULE.

Reduce the price to guineas; then for every guinea reckon a penny, and a farthing for a quarter guinea.

Ex.

At 97 l. *per ton, what is that a gallon?*

97 l. is	92 g. 8 s.		
92 pence	—	7	8
8 ſhillings	—		0¼
Anſ.		7	8¼

PROB. XIII.

Knowing the price of 1 *pound, to find what the hundred weight coſts.*

1 RULE.

For every penny which a pound denotes,
Count once the angels, 'bating twice the groats.

Ex.

If a pound be $7\frac{1}{2}$ d. *what is the hundred weight?*

$7\frac{1}{2}$ angels	3 l.	15 s.
15 groats, ſub.		5
Anſ.	3	10

2 RULE.

Number the farthings that each pound denotes,
Take twice the ſhillings, once as many groats.

Ex.

If 1 *coſt* 3 s. $6\frac{3}{4}$ d. *what will* 100 *weight coſt?*

3 s. $6\frac{3}{4}$ d. is 171 farthings.

twice 171 ſhillings	17	2	0
171 groats, add	2	17	0
	19	19	0

3 RULE.

Find the pence for 1; take 9 times as many ſhillings, and once as many groats.

Ex.

At 10 d. *per pound, what is the hundred weight?*

9 ten pences		90	
ten groats		3	4
		93	4
Anſ.	4	13	4

PROB.

PROB. XIV.

The price of a hundred weight being given, to find that of a pound?

1 RULE.

For every 7 ſhillings in the price of a hundred weight, reckon 3 farthings for a pound.

Ex.

At 8 *l. a hundred weight, what will one coſt?*

8 pounds are	160 ſhillings
the 7th part	23
3 times 23 farth^s.	69 = 17¼ pence, Anſw.

2 RULE.

Take the tenth part of the price in ſhillings, and the fourteenth part of that; the ſum is the pence for a pound.

Ex.

At 29 *l.* 17 *s.* 6 *d. the hundred, how much is a pound?*

29 l. 17 s. 6 d. is	597½ ſhillings.
the tenth part	59.7
$\frac{1}{14}$ of this —	4.2
ſum	64 pence.
Anſ.	5 s. 4 d.

PROB. XV.

In Troy weight, having the price of a grain, to find the price of an ounce, or a pound?

RULE.

Multiply the farthings in the price, by 6; gives the price of a pound, in pounds.

Or double the pence in the price; gives the price of an ounce in pounds ſterling.

Ex. 1.

At $1\frac{1}{4}$ d. *a grain, how much is a pound?*
$1\frac{1}{4}$ or $5^{f} \times 6 = 30$ l. Anſ.

Ex. 2.

At 2 *d. a grain, how much is an ounce of gold?*
2 d. doubled is 4 pounds. Anſ.

PROB. XVI.

In Troy weight, the price of a penny weight being given, to find that of a pound.

RULE.

The number of pence in the price, call pounds.

Ex.

And $10\frac{1}{2}$ *d. the penny weight, how much is a pound?*
$10\frac{1}{2}$ pounds 10 l. 10 s. Anſw.

PROB. XVII.

The price of a buſhel of coals being given, to find that of a chaldron, or 36 *buſhels.*

RULE.

Thrice the number of pence in the price, call ſhillings.

Ex.

At $4\frac{1}{2}$ *d. the buſhel, how much is a chaldron of coals?*
Thrice $4\frac{1}{2}$ is $13\frac{1}{2}$ ſhillings, or 13 s. 6 d. Anſ.

PROB.

PROB. XVIII.

Given the price of a ſtone, to find the price of a ton.

RULE.

Find the pence, and take $\frac{2}{3}$ that number of pounds.

Ex.

At 3 *s.* $4\frac{1}{4}$ *d. a ſtone, what is that a ton?*

3 *s.* $4\frac{1}{4}$ *d.* is $40\frac{1}{4}$.

and $\frac{2}{3}$ of $40\frac{1}{4}$ is $\frac{80}{3} + \frac{1}{6}$ or $26\frac{5}{6}$ l.

and $26\frac{5}{6}$ l. is 26 l. 16 s. 8 d. Anſ.

PROB. XIX.

The price of one thing being given in pence, to find what any number will come to.

1 RULE.

For 6 pence reckon half ſo many ſhillings;
For 3 pence, half of the laſt ſum; and
For a penny, $\frac{1}{3}$ of the laſt.

Example.

What will 57 *coſt at* 8 d. *a piece?*

	s.	d.	l.	s.	d.
half of 57	28	6	1	8	6
half of this	14	3		0	0
a third of that	4	9		4	9
ditto —	4	9		4	9
Anſ.			1	18	0

2 RULE. *By a Table.*

Againſt the quantity on the ſide, and under the price of 1 at top, you have the price of your quantity. This is to be taken out at ſeveral times as occaſion requires.

num	1 farth.			2 farth.			3 farth.			1pen.			2 pen.			3 pen.			4 pen.		
	l.	s.	d.	l.	s.	d.	l.	s.	d.												
1			0¼			0½			0¾			1			2			3			4
2			0½			1			1½			2			4			6			8
3			0¾			1½			2¼			3			6			9		1	0
4			1			2			3			4			8		1	0		1	4
5			1¼			2½			3¾			5			10		1	3		1	8
6			1½			3			4½			6		1	0		1	6		2	0
7			1¾			3½			5¼			7		1	2		1	9		2	4
8			2			4			6			8		1	4		2	0		2	8
9			2¼			4½			6¾			9		1	6		2	3		3	0
10			2½			5			7½			10		1	8		2	6		3	4
20			5			10		1	3		1	8		3	4		5	0		6	8
30			7½		1	3		1	10½		2	6		5	0		7	6		10	0
40			10		1	8		2	6		3	4		6	8		10	0		13	4
50		1	0½		2	1		3	1½		4	2		8	4		12	6		16	8
60		1	3		2	6		3	9		5	0		10	0		15	0	1	0	0
70		1	5½		2	11		4	4½		5	10		11	8		17	6	1	3	4
80		1	8		3	4		5	0		6	8		13	4	1	0	0	1	6	8
90		1	10½		3	9		5	7½		7	6		15	0	1	2	6	1	10	0
100		2	1		4	2		6	3		8	4		16	8	1	5	0	1	13	4
200		4	2		8	4		12	6		16	8	1	13	4	2	10	0	3	6	8
300		6	3		12	6		18	9	1	5	0	2	10	0	3	15	0	5	0	0
400		8	4		16	8	1	5	0	1	13	4	3	6	8	5	0	0	6	13	4
500		10	5	1	0	10	1	11	3	2	1	8	4	3	4	6	5	0	8	6	8
600		12	6	1	5	0	1	17	6	2	10	0	5	0	0	7	10	0	10	0	0
700		14	7	1	9	2	2	3	9	2	18	4	5	16	8	8	15	0	11	13	4
800		16	8	1	13	4	2	10	0	3	6	8	6	13	4	10	0	0	13	6	8
900		18	9	1	17	6	2	16	3	3	15	0	7	10	0	11	5	0	15	0	0
1000	1	0	10	2	1	8	3	2	6	4	3	4	8	6	8	12	10	0	16	13	4

Examp.

	5 pen.			6 pen.		
1			5			6
2			10		1	0
3		1	3		1	6
4		1	8		2	0
5		2	1		2	6
6		2	6		3	0
7		2	11		3	6
8		3	4		4	0
9		3	9		4	6
10		4	2		5	0
20		8	4		10	0
30		12	6		15	0
40		16	8	1	0	0
50	1	0	10	1	5	0
60	1	5	0	1	10	0
70	1	9	2	1	15	0
80	1	13	4	2	0	0
90	1	17	6	2	5	0
100	2	1	8	2	10	0
200	4	3	4	5	0	0
300	6	5	0	7	10	0
400	8	6	8	10	0	0
500	10	8	4	12	10	0
600	12	10	0	15	0	0
700	14	11	8	17	10	0
800	16	13	4	20	0	0
900	18	15	0	22	10	0
1000	20	16	8	25	0	0

Examp.

What comes 1453 *to, at* 9½ *d. a piece?*

	l.	*s.*	*d.*
1000 (at 6d.)	25	0	0
at 3d.	12	10	0
at ½	2	1	8
400 (at 6 d.)	10	0	0
at 3 d.	5	0	0
at ½		16	8
50 (at 6 d.)	1	5	0
at 3 d.		12	6
at ½		2	1
3 (at 6 d.)		1	6
at 3 d.			9
at ½.			1½
Anſ.	57	10	3½

PROB. XX.

The price of 1 *being given in ſhillings, to find what any number will come to.*

1 RULE.

In the number of things propoſed, double the right hand figure for ſhillings, and reckon the reſt of it pounds. Then take this ſum of pounds and ſhillings as oft as 2 is contained in the price; and if there is an odd ſhilling, take half the ſaid ſum.

Examp.

What is 2344 *ounces at* 13 *ſhillings an ounce?*

234 l. 8 s. taken 6 times is	1406 l.	8 s.
half of it —	117	4
Anſ.	1523	12

2 RULE.

Multiply the number of things by half the price; then double the right hand figure for ſhillings, and the reſt is pounds.

In the former Example.

2344 × by $6\frac{1}{2}$ is as follows,

6 × 2344 is	14064	
half of it is	1172	
	15236	
Anſ.	1523 l.	12 s.

PROB. XXI.

Having the price of 1 in pounds; to find the price of any number of them.

RULE.

Multiply the price ſeparately by every figure of the propoſed number; putting each product in its proper place.

Examp.

What comes 5307 *to at* 57 l. *a piece?*

5000 × 57 — 285000
300 × 57 — 17100
7 × 57 — 399
Anſ. 302499 l.

PROB. XXII.

To caſt up ſums of money by Counters.

RULE.

Make four ſeparate partitions upon a table, and mark them with chalk, pounds, ſhillings, pence, farthings; then write down the ſeveral ſums, or elſe put as many counters into each partition as there are pounds, ſhillings, &c. in each ſum. Then begin to reckon at the loweſt denomination, and proceed till you amount to 1 of the next greater; then throw away that number of counters in the leſſer, and inſtead of them, put 1 counter in the place of the greater. Proceed thus throughout, leaving only ſo many counters in each partition, as is the overplus at laſt.

Note, pieces of paper may ſerve inſtead of counters, and brown paper may ſerve for pence, and white, for ſhillings.

Example.

Suppoſe you have the following ſums,

l.	*s.*	*d.*
3	7	8
105	12	$2\frac{1}{2}$
17	11	$6\frac{3}{4}$
9	0	11

	poun.	ſhill	penc.	far.
	o	o	o	
		o		
tot.	135	oooo	oo	o
	ooo	oooo	oo	
	oo	oooo		
	oo			
	o			
	o			

Firſt, I ſay, 3 farthings and a halfpenny are 5 farthings, leave 1 counter in the place of farthings; and carry 1 to the place of pence. Then 1 and 11 is 12, which throw away, and put 1 counter in the place of ſhillings; then 6 and 2 is 8 and 8 is 16, throw away 12, and put 1 in the place of ſhillings, and put in 4 counters at pence. Then 2 and 11 is 13 and 12 is 25, throw away 20, and but 1 in the place of pounds. Then 5 and 7 is 12, ſo put 12 counters in the place of ſhillings. Laſtly, 1 and 9 is 10 and 7 is 17, and 5 is 22, and 3 is 25, and 10 is 35, and 100 is 135; ſo put 135 counters in the place of pounds. So examining each partition, you will find 135 l. 12 s. $4\frac{1}{4}$ d. to be the ſum required.

PROB. XXIII.

To find the intereſt of a ſum of money for any number of days, at any rate per cent.

RULE.

Multiply the rate per cent. and the principal together, and take the third part of it, from which cut off 3 places to the right hand. From this ſubtract the 73d part of itſelf (or for ſhortneſs the 70th part); and the remainder is the intereſt for 12 days.

Then 2, 5, 10 times this, &c. gives the intereſt for 24, 60, 120 days, &c. Alſo $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ the ſaid intereſt gives the intereſt for 1, 2, 3, 4, 6 days; and conſequently it will be had for any number of days.

Examp.

What is the intereſt of 311 l. 13 s. 4 d. for 315 days, at $4\frac{1}{2}$ per cent.

311 l. 13 s. 4 d. = $311\frac{2}{3}$ =	311.66666
$4\frac{1}{2} \times 311.666$ is —	1402.5
the third part is —	467.5
three places cut off —	.4675
the 70th part, ſubſt.	66
the intereſt for 12 days	.4609
Intereſt for 120 days —	4.609
—— for 120 days more	4.609
—— for 60 days more	2.3045
—— for 12 days more	.461
—— for 3 days more	.1152
315	12.0987
	20
	1.9740
	12
	11.6880

Anſ. 12 l. 1 s. $11\frac{3}{4}$ d.

PROB. XXIV.

Having your weekly expences; to know what it coſts you in the year.

1 RULE.

Each ſix-pence in the week does make appear,
The guineas with the crowns ſpent in a year.

Examp.

At 5 s. 6 d. *per week, or* 11 *ſix-pences per week.*

		l.	s.	d.
11 guineas	—	11	11	0
11 crowns	—	2	15	0
	Anſ.	14	6	0

2 RULE.

The farthings in each week do make appear,
The ſhillings with the pence ſpent in a year.
Add one day's expence.

Ex.

At 15¼*d. in a week, what is it in a year?*
15¼ is 61 farthings.

		l.	s.	d.
61 ſhillings	-	3	1	0
61 pence			5	1
One day	—			2¼
		3	6	3¼

PROB. XXV.

Knowing your daily expences; to find what it coſts in a year.

RULE.

Number the pennies of each day's expence;
So many pounds, and angels, groats, and pence,
You ſpend within the year's circumference.

Examp.

At 9d. *per day, what for the year?*

		l.	s.	d.
9 pounds	—	9	0	0
9 angels	—	4	10	0
9 groats	—		3	0
9 pence	— —			9
	Anſw.	13	13	9

PROB. XXVI.

By knowing your yearly income; to find what you can ſpend a day.

RULE.

The third part of the pounds you yearly have,
So many two-pences afford to ſpend or ſave. nearly.

Examp.

At 40 l. *a year, quere what a day?*

$\frac{1}{3}$ of 40 is $13\frac{1}{3}$
and $13\frac{1}{3}$ two-pences is $26\frac{2}{3}$ pence, or 2 s. $2\frac{1}{2}$ d.

PROB. XXVII.

The money gained in an hour being given; to find the produce in a week.

RULE.

Reduce the price to pence, and take $\frac{7}{10}$ thereof for the pounds.

Examp.

If an hour produces $9\frac{1}{2}$d. what will be the product in a week?

Here $9\frac{1}{2} \times \frac{7}{10}$ is 6.65 = 6 l. 13 s. Anſw.

PROB. XXVIII.

Having the product of any thing for a minute, to find it for a day.

RULE.

Take ſo many pounds, and ſo many angels as there are farthings in the product of one minute.

Examp.

At $5\frac{1}{4}$ d. a minute, what will a day come to?

$5\frac{1}{4}$ d. is 21 farthings.

		l.	s.	d.
21 pounds		21	0	0
21 angels	—	10	10	0
	Anſ.	31	10	0

PROB. XXIX.

To diſcover a deceitful Ballance.

RULE.

After the weights are made even in the two ſcales, change them into the oppoſite ones, Then if they ſtill remain equal, the ballance is true; if not, 'tis falſe.

But a ſteelyard cannot be tried this way; yet it may be tried, by weighing any thing with it, and then weighing that thing in a true pair of ſcales.

Cor. *If one arm of a ballance be longer than the other it is a deceitful ballance; though the ſcales, when empty, may be in equilibrio.*

PROB. XXX.

To weigh all things with the feweſt weights.

RULE.

Take any number of pound weights in this progreſſion 1, 3, 9, 27, 81, &c. and they will weigh any number of pounds, not exceeding the ſum of them all. But ſometimes ſome of the weights muſt be put in the contrary ſcale, to be ſubtracted.

The ſame thing may alſo be done with the weights in this progreſſion, 1, 2, 4, 8, 16, 32, &c. where none need be put in the oppoſite ſcale, which is better.

PROB. XXXI.

To find the content of any plane figure by weighing it.

RULE.

Having the figure drawn upon paper, cut it out in paſteboard, which is eaſily done, by laying the paper ſcheme upon it; and weigh your paſteboard figure in grains. Then mark upon it any ſquare you can meaſure, and cut it all away to that ſquare; and then weigh the ſquare. And ſay, as the weight of the ſquare, to its meaſure; ſo is the weight of the figure, to its content in the ſame meaſure.

PROB. XXXII.

The ſolid content of a body being known, to find its weight.

RULE.

Find what an inch or a foot of that ſort of body weighs, either by trial, or by the tables of ſpecific gravity. Then multiply the content of the body, by this weight, and you have the weight of the whole.

Ex.

How much does a piece of elm weigh, which contains 20 *feet?*

By the table of ſpecific gravities, a foot of elm weighs 49 l. 1 oz. This multiplied by 20, produces 981¼ lb. that is, 70 ſt. 1¼ lb. or 8 hundred 6 ſtone, 1¼ lb.

PART II.

Geometrical Problems, and the Meaſuring of Lines.

SINCE meaſuring the contents of ſuperficies and ſolids by the Sliding rule, is very eaſy and expeditious; and we ſhall have occaſion to make frequent uſe of it, in what follows; therefore I ſhall begin with a ſhort deſcription thereof.

The deſcription and uſe of the Sliding rule.

This Rule (commonly called *Cogſhal's* Sliding rule) has four lines upon it, adapted to meaſuring; they are marked A, B, C, D. Three of theſe, A, B, C, are double lines, becauſe they proceed from 1 to 10 twice over. The fourth line D is a ſingle line, proceeding from 4 to 40, and is called the *Girt line.*

Two of theſe lines B, C, are upon the ſlider, and the other two A, D, upon the rule. The three ſingle lines are all exactly alike, and alike numbered, and divided; the diviſions being numbered 1, 2, 3, 4, to 1 again, from the beginning to the middle; and again, 1, 2, 3, 4, to 10, from the middle to the end. The fourth line D, is numbered 4, 5, 6, 7 to 40; but figures are not ſet to all the diviſions, for towards the end they grow very ſmall. The number 4 in the ſingle line ſtands againſt 1 in the double lines, and the number 40 againſt 10. In all the lines the diviſions are large at the beginning, and continually decreaſe to the laſt.

For the numeration upon the lines; 1 at the beginning may be counted 1, then the 1 in the middle will be 10, and 10 at the end 100. If 1 at the beginning be called 10, 1 in the middle is 100, and 10 at the end 1000. Or if the firſt 1 be reckoned 100, the ſecond 1 will be 1000, and the laſt 1 or the 10 will be 10,000; and ſo on. And as the grand diviſions are numbered or altered, ſo are all the ſmaller diviſions altered in value accordingly.

The uſe of the double lines A and B, is for working the rule of proportion; as alſo for finding the areas of plain figures. And the uſe of the girt line D, and the other double line C, is for meaſuring ſolids.

Beſides theſe lines, there are ſeveral other lines and ſcales, as a line of inches with halves, quarters, and half quarters. A line of inches, each divided into ten parts; this is alſo a foot divided decimally, and ſerves to turn inches into decimal parts of a foot. The backſide of the ſliding piece is divided into inches, halves, quarters, and half quarters; and ſerves to meaſure any length, by ſliding it out.

There are upon ſome of theſe rules other ſcales and tables, as a table of board meaſure, one of timber meaſure. A line ſhewing what length, for any breadth, will make a foot ſquare: this is for compleating the table of board meaſure. Alſo a line, ſhewing what length will make a ſolid foot, for any thickneſs. This alſo compleats the table of ſolid meaſure.

If 1 on A be ſet to any number on B, theſe two lines will be a table of multiplication. For the numbers in A, will be the ſeveral multiplicands, and the correſponding numbers in B, the products.

Alſo if the lines C, D, be ſet together, 10 againſt 10; they will be a table of ſquares. The line D being the roots, and C the ſquares; the numbers at 10 and 10 being reckoned 10 and 100.

In a Gauger's rule, the line D is numbered from 1 at the beginning to 10 at the end; which is more commodious for extracting roots.

As the ſhorteſt and eaſieſt way of meaſuring all ſuperficies and ſolids, is by the ſliding rule; it will be neceſſary to lay down a few Problems to ſhew the uſe of it.

The Uſe of the SLIDING RULE.

PROB. 1.

To multiply two numbers together, as 7 and 26.

RULE.

Set 1 upon A, to one of the numbers (26) upon B; then againſt the other number (7) on A, is found the product (182) upon B.

Note, If the third term on A runs beyond the end of the line B, ſeek that third term on the other radius (or part) of the line A; then the product will be found on B, which muſt be increaſed ten times as much.

PROB. 2.

To divide one number by another, as 510 by 12.

RULE.

Set the diviſor (12) on A, to 1 on B; then againſt the dividend (510) on A, is the quotient ($42\frac{1}{2}$) on B.

Note, If the dividend runs beyond the end of the line A, diminiſh it 10 or 100 times, to make it fall on A; then increaſe the quotient 10 or 100 times, reſpectively.

PROB. 3.

To ſquare any number, as 27.

RULE.

Set 1 (or 10) upon D to 1 upon C, then againſt the number (27) upon D, is the ſquare upon C (729). And to reckon right, obſerve that as the 1 upon D is counted 1, 10, or 100, &c. the 1 upon C (againſt it) will be 1, 100, or 10,000; that is the ſquare of the other.

If you would ſquare 270, reckon the 1 on D to be 100, then that on C will be 10,000, and the product 72,900.

PROB. 4.

To extract the ſquare root of any number, as 4268.

RULE.

Set 1 upon C, to 1 (or 10) upon D; then againſt the number (4268) on C, is the root on D (65·3). Which to value right, you muſt ſuppoſe the 1 on C to be ſome of theſe ſquares 1, 100, 10,000, &c. which is near the given number; and then the root correſponding, will be the value of the 1 on D.

PROB. 5.

To find a mean proportional between any two numbers, as 27 and 450.

RULE.

Set one of the numbers (27) on C, to the ſame on D; then againſt the other numbers (450) on C, you have the mean (112) on D.

Note, if one of the numbers overruns the line, take the 100th part of it, and then take a mean, which multiply by 10.

PROB. 6.

Three numbers being given; to find a fourth proportional: ſuppoſe 12, 28, and 57.

RULE.

Set the firſt (12) upon A, to the ſecond (28) upon B; and againſt the third (57) on A, is the fourth (133) on B.

Note, if one of the middle numbers runs off the line, take the 10th part of it only, and then take 10 times the fourth proportional found. The finding a third proportional is the ſame, the ſecond number being twice repeated.

PROB. 7.

If the breadth of a board be given (8 *inches*); *to find how much in length will make a foot ſquare.*

RULE.

If the board be narrow, you will find it in the table of under board meaſure in the table on the rule; if not, ſhut the rule, and ſeek the breadth (8) in the line of board meaſure running along the rule, from that table; then over againſt it, on the oppoſite ſide, in the ſcale of inches, is 18 inches, the length required.

PROB. 8.

The ſide of the ſquare of a piece of timber being given (16 inches); to find how much in length will make a foot ſolid.

RULE.

If the timber be ſmall, look into the table of under timber meaſure on the rule, and you'll find the feet and inches. Otherwiſe look for the ſide of the ſquare (16) in the line of timber meaſure running along the rule from that table: and againſt it, in the line of inches, is the length 6$\frac{3}{4}$ inches.

PROB. 9.

To find the content of any ſurface.

RULE.

Reduce the ſurface to a rectangle, then (by Prob. I.) multiply the length and breadth together, for the area or content of the ſurface, in the ſame ſort of meaſure.

Otherwiſe.

If the length be given in feet, and the breadth in inches. Set 12 on A to the length on B; and againſt the breadth on A, is the content on B, in feet.

Otherwiſe.

If you would have the dimenſions taken in one ſort of meaſure, and the area given in another ſort. See how many ſquares of the firſt, make one of the other; and make it a diviſor.

Then ſet this diviſor on A to the length on B, and againſt the breadth on A, is the area on B.

PROB.

PROB. 10.

To find the content of the ſolid body.

RULE.

Reduce it to a long body, or parallelopipedon, ſquare at the ends; meaſure the length in feet, and the ſide of the ſquare in inches. Then ſet 12 on D, to the length on C; and againſt the ſide of the ſquare on D, is the content on C, in feet.

PROB. 11.

If two fixt numbers are given on the lines D *and* C, *as p and q; to change them to others that will perform the ſame operations.*

RULE.

Inſtead of p and q, take 1 and $\frac{q}{pp}$, or $\sqrt{\frac{pp}{q}}$ and 1, or r and $\frac{qrr}{pp}$; which may be expreſſed thus,

D	C
p	q
1	$\frac{q}{pp}$
$\sqrt{\frac{pp}{q}}$	1
r	$\frac{qrr}{pp}$.

That is, inſtead of ſetting p upon D, to q upon C; one may ſet 1 on D, to $\frac{q}{pp}$ on C; or $\sqrt{\frac{pp}{q}}$ on D, to 1 on C; or r on D to $\frac{qrr}{pp}$ on C.

For, p, q, being two fixt numbers, and r take at pleaſure; and ſince the numbers on C are as th ſquares of thoſe on D; therefore we muſt ſquare thoſe on D, and make them proportional, that they may perform the ſame thing; then we ſhall have $pp : q :: rr : s$, and $s = \frac{qrr}{pp}$, and $r = \sqrt{\frac{pps}{q}}$.

PROB. 12.

To find a proper fixt number for the line D, *for meaſuring planes or ſolids.*

RULE.

Here you muſt have given the content of ſome particular plane or ſolid of that ſort, whoſe diameter, circumference, &c. is 1, put this content = S.

Then if the dimenſions be taken in one meaſure, and the content given in another; put n = number of leſſer ſquares or cubes contained in 1 of the greater. x = fixt number. Then $x = \sqrt{\frac{n}{S}}$.

For in areas it will be $x^2 : 1 :: 1 : S$, and $x = \sqrt{\frac{1}{S}}$.

And in ſolids, $x^2 \times \text{height} : \text{height} :: 1 : S$. And $x = \sqrt{\frac{1}{S}}$.

Examp.

In ſquares $x = 1$. Or for inches and feet, $x = 12$.

In circles $x = \sqrt{\frac{1}{.7854}}$. Or for inches and feet, $x = \sqrt{\frac{144}{.7854}}$.

In circles, when the circumference is 1, $x = \sqrt{\frac{1}{.0796}}$.

In cylinders for inches and feet, $x = \sqrt{\frac{144}{.7854}}$.

In a cube, $x = 1$. For inches and feet, $x = \sqrt{\frac{144}{1}} = 12$.

In a ſphere, circumference $= 1$, for inches and feet, $x = \sqrt{\frac{1728}{.0169}}$, &c.

PROB. I.

To draw a right line from one place to another.

RULE.

If it be a ſmall diſtance, apply a ruler, to the two points the line is to paſs through, and by the edge of it draw a line with a pin, or pencil, or with pen and ink.

If the diſtance be great, ſtretch a line from one place to the other. And if it be a plane chalk the line, and cauſe it to ſpring againſt that plane, and it will leave the mark of a right line. If ſtakes be placed all the way between the eye and any diſtant point, they will ſtand in a right line.

PROB. II.

To meaſure the length of any right line.

RULE.

If the diſtance be ſhort, open a pair of compaſſes to the extent of the line, and apply it to a ſcale of equal parts, and you will have the length in the ſame ſort of parts.

If the diſtance be great, apply any meaſure, as a foot, yard, &c. to one end of the line, and extend it as far as it will reach directly forward; do thus as often as you can, till you come to the other end;

Fig then ſo many times as you applied that meaſure, ſo many feet or yards your line is of.

Or extend a line from one end to the other; and meaſure the length of that line, by applying any meaſure to it, as oft as you can; and you will find how many of theſe meaſures it contains.

In taking the dimenſions of any ſurface or ſolid, they muſt all be taken by one ſort of meaſure; all feet, all yards, &c.

PROB. III.

To draw one line parallel to another, thro' any point.

RULE.

1. Let AB be the line and D the point; from the point D take the neareſt diſtance to the line AB, with a pair of compaſſes, or with a thread. And from ſome other point A in the line AB, ſet off the ſame diſtance to C, and deſcribe a ſmall arch at C. Draw a line from D to touch the arch at C; then CD is parallel to AB.

SCHOLIUM.

Theſe parallels are ſuppoſed both of them to be drawn upon ſome plane ſurface. But if one of them, as CD, be ſuppoſed to be in the air; beſides the equal diſtances, care muſt be taken, that they both be in the ſame plane; and that may be known, by applying the eye to one of them, as CD, for if it cover the other AB, then they are in one plane, otherwiſe not.

PROB.

PROB. IV.

To raiſe a perpendicular to a right line, to paſs thro' a given point.

RULE.

Apply a ruler to that line, then apply one ſide of a ſquare to the ruler, ſo that the other ſide may paſs thro' the point; or elſe that another rule applied thereto may paſs thro' the point; then draw a line by the ſide of the ſquare or the ruler.

Or draw a thread from that point, ſo that it may coincide with the ſide of the ſquare, while the other ſide lies upon, or coincides with the given line; and the thread will be a perpendicular.

Otherwiſe.

Make the baſe AB, 4; the height BC, 3, and, the ſide AC, 5, feet, yards, &c. and then CB is perpendicular to AB. 2.

PROB. V.

To raiſe a perpendicular upon a plane to paſs thro' any point.

RULE.

Extend a thread from the point to the plane, and put it in ſuch a poſition, that it may juſt coincide with one ſide of a ſquare, when the other ſide is laid on the plane, round about the thread on every ſide. Then the thread is a perpendicular to the plane.

Or extend a thread from the point to three diſtinct places on the plane. Find the centre of a circle paſſing thro' thoſe three points, which is ſoon done by a few trials: then a line, paſſing through that center and the given point, is a perpendicular.

Fig. 2. Or a circle may be deſcribed on the plane with a pencil, by extending the thread; and then the center of this circle may be found, by drawing two parallel lines thro' it, and drawing a third line thro' the middle of theſe two; for if that line be biſected, the middle point will be the center.

PROB. VI.

To meaſure the perpendicular height of any point above a line or plane.

RULE.

Take the neareſt diſtance from that point to the line or plane, with a ſtring; and then meaſure its length.

If the perpendicular falls within a ſolid body; you muſt lay a ruler from that point, parallel to the baſe, and take the neareſt diſtance from the ruler to the baſe; and this is eaſily done, if the body ſtands upon a plain floor; but if not, then a board muſt be applied to the plane of its baſe, and a line drawn parallel to the board through the top of the body, which muſt be held ſo as to be every where equidiſtant; and then that diſtance is the height required.

If it be ſo ſituated, that a line or ruler cannot be applied, ſo as to be made parallel to the baſe. Then if it has a ſtreight ſide, the angle that the ſide makes with the baſe, and the length of the ſide, muſt be found; from whence the perpendicular height may be had.

PROB. VII.

To divide a line AB *into any number of equal parts.*

RULE.

Draw any line *ab* parallel to AB, and ſet off your number of any equal parts from *a* to *b*. Draw the lines *a*A and *b*B, to interſect in C. From C draw C*f*, C*e*, C*d*, C*c*; to interſect AB in 1, 2, 3, 4; which are the diviſions required. 3.

The line AB may alſo be divided by trials, by running a pair of compaſſes from one end to the other; and increaſing or decreaſing the diviſions, till they juſt anſwer to the whole.

If the diſtance be large, this way cannot be practiſed; then the length of the line AB muſt be meaſured in feet, yards, &c. and the whole length divided by the number of parts, to get the length of one part; which done, that length muſt be ſet off from B to 1, from 1 to 2, from 2 to 3, &c. to A.

PROB. VIII.

To deſcribe a circle thro' three points given, A, B, F.

RULE.

Open the compaſſes, and ſetting one foot in C, try if the other turned about will reach all the three points; if not, ſhift the center, and try again, till it do: or you may uſe a thread; then deſcribe the circle ABF. 4.

Otherwiſe.

From two of the points draw lines to the third, as AB, BF. Raiſe two perpendiculars DC, EC, upon the middle of the lines AB, BF; to interſect in C. From the center C deſcribe the circle ABF.

Fig.

PROB. IX.

To find the center of a circle.

1 RULE.

4. The center C may be found by trials; or thus; draw any line BF, and on the middle point E, erect the perpendicular GEH. Biſect GH in C, and C is the center.

2 *Or thus.*

Thro' the circle, draw any two lines BF, *bf*, parallel to one another; biſect theſe lines at E and *e*, thro' E, *e*, draw the line GH, which will be a diameter; therefore if GH be biſected in C, then C will be the center.

PROB. X.

To divide the ſemicircle ADB *into any number of equal parts.*

RULE.

5. Make AC and BC equal to AB. Divide AB into the ſame number of equal parts, at *b*, *c*, *d*, *e*. And let the diviſions at the center rather be bigger than leſs than thoſe near the circumference at A and B. Then draw the lines C*b*, C*c*, C*d*, C*e*, to interſect the circle at 1, 2, 3, 4, which will divide the ſemicircumference into equal parts as required.

The ſemicircle, or any other arch, may alſo be divided into any number of equal parts by trials, running a pair of compaſſes from one end to the other, and increaſing or decreaſing the extent, as it is found to be too little or too big. Where note, the length of the radius is juſt the extent of the third part of the ſemicircle ADB.

Fig.

PROB. XI.

To make a ſquare equal to a given circle.

1 RULE.

Thro' the center C draw two diameters AB, DE perpendicular to one another. And divide each of the quadrants into three equal parts at 1, 2. So that the middle diviſion of each be rather bigger, than leſs, than the other two. Thro' 1, 1, draw the lines *ab*, *cd*; and thro 2, 2, the lines *ac*, *bd*; and the ſquare *abdc* is equal to the circle AEBD. 6.

2 *Otherwiſe.*

Thro' the center draw two diameters AC, BD, perpendicular to each other. And divide one diameter AC into eight equal parts, and ſet one of theſe parts (rather more than leſs) from A to *a*, and from B to *b*, C to *c*, and D to *d*. Draw *ab*, *bc*, *cd*, *da*. And *abcd* is a ſquare equal to the circle ABCD. 7.

3 *Otherwiſe.*

Make the diameter AB 14 parts, and AD 11; raiſe the perpendicular DE. Draw AE, and the ſquare AEFG is equal to the whole circle, whoſe diameter is AB. 9.

By the Sliding Rule.

Set 1 on A to .282 on B, and againſt the circumference on A, is the ſide of the ſquare on B; whoſe area is equal to the circle.

For .282 is the ſide of a ſquare equal to the circle, whoſe circumference is 1.

Fig.

PROB. XII.

To deſcribe an Oval.

RULE.

8. Draw the tranſverſe AB, and conjugate CD, at right angles to it. Deſcribe two circles AFEK, and EGBL; then take two centers H, I, any way in D, equidiſtant from E; and from theſe centers deſcribe the arches KL, MN, to touch the circles KF and GL.

The two circles may be drawn otherwiſe than to touch at E; for they may be made to paſs thro' each other's center, or at a diſtance from one another, &c.

An ellipſis is eaſily drawn, by ſetting up two pins in two centers *n*, *o*; about which putting a thread *nop*, and knotting it at *p*; the point *p* carried about, will deſcribe an ellipſis DPKAMCNBLD, by help of a pencil.

PROB. XIII.

To reduce any irregular figure, or any curve or part of a curve, into a rectangle.

RULE.

10. Let ACB be the figure. Erect any number of perpendiculars upon the baſe AB at equal diſtances; meaſure their lengths, and add them together, and divide the ſum by their number, the quotient is the mean breadth AD, which is the breadth of the parallelogram ADEB, equal to the fig. ACB: the length being the ſame; or both of them having the common baſe AB.

Examp.

Fig. 10.

Example.

Erect ſeven perpendiculars, whoſe lengths meaſured and marked in the figure, are 0, 2, 4, 4, 2, 3, 2. Their ſum is 17, and 17 divided by 7, gives $2\frac{3}{7}$ for the mean breadth, which is equal to BE or AD, the breadth of the parallelogram ABED, equal to the figure ACB.

Note, this Prob. is of uſe in meaſuring irregular planks or boards.

PROB. XIV.

To find the hypothenuſe of a right angled triangle.

RULE.

From any ſcale lay down the length of the baſe AB. Draw BC perp. to it, of the length given, by the ſame ſcale. Draw AC, which, meaſured on the ſcale, gives its length. 11.

Otherwiſe.

To the ſquare of the baſe add the ſquare of the perpendicular; extract the ſquare root of the ſum, and you have the hypothenuſe.

Examp.

Let the baſe AB *be* 22, *the perpendicular* BC 13.

22	13	653(25·55	the hypothe-
22	13	4	nuſe AC.
44	39	45)253	
44	13	225	
484	169	505) 2800	
	484	2525	
	653	275	

PROB.

PROB. XV.

To find either leg of a right angled triangle.

1 RULE.

11. Draw the given ſide AB, of it due length; raiſe BC perpend. to it; with the length of the hyp. in your compaſſes, ſet one foot in A, and with the other deſcribe an arch cutting CB in C; then CB is the other ſide, which meaſured on the ſcale, gives its length.

2 *Or thus.*

From the ſquare of the hyp. ſubtract the ſquare of the given ſide; extract the ſquare root of the remainder, and you have the other ſide.

Examp.

Let the hypothenuſe AC *be* 25.55, *the baſe* AB 22.

```
                 . .         . .
  22           25.55        168(12.96, the perpen-
  22           25.55        1         dicular, CB.
 ----          -----       ----
   44           5110       22)68
  44            1277          44
 ----            128       -----
  484             13     249)2400
               -----          2241
               652.8         -----
               484             159
               -----
               168.8
               -----
```

PROB. XVI.

Having the three ſides of any triangle ACB, *to find the perpendicular* CD.

1 RULE.

Draw the baſe AB of a proper length, by any ſcale; and with the length of AC in your compaſſes, and one foot in A, deſcribe a ſmall arch; then with BC in your compaſſes, and one foot in B, croſs that arch at C. Then draw AC, BC; and the triangle is conſtructed. 14. 15.

Then from C let fall a perpendicular upon AB, as CD, for the perpendicular. Or take the neareſt diſtance from C to AB, which apply to your ſcale.

2 RULE.

Find one ſegment as $AD = \frac{AC^2 + AB^2 - CB^2}{2AB}$.

Then find $CD = \sqrt{CA^2 - AD^2}$.

3 RULE.

Take half the ſum of the three ſides, and ſubtract each ſide from it, and you have three remainders; then multiply that half ſum, and the three remainders together continually. Extract the ſquare root, and divide it by half the baſe, and it gives the perpendicular.

Cor. *Hence either ſegment of the baſe is had thus*; To the ſquare of the adjoining ſide add the ſquare of the baſe, from which ſubtract the ſquare of the oppoſite ſide; and then divide by twice the baſe.

PROB.

Fig.

PROB. XVII.

To find the circumference of a circle, from the diameter.

1 RULE.

23. Say as 7 to 22, ſo the diameter to the circumference.

2 *Or more exactly.*

As 113 to 355, ſo the diameter to the circumference.

3 *Or thus.*

Multiply the diameter by 3.1416, the product is the circumference.

Examp.

Suppoſe the diameter be 32.

	Or thus.
7 : 22 : : 32 :	3.1416
22	32
64	62832
64	94248
7)704(100.6	100.5312

1 *By the Sliding Rule.*

Set 7 on A, to 22 on B, then againſt the diameter (32) on A, is the circumference (100.6) on B.

2 *Or thus.*

Set 1 on A, to 3.14 on B; then againſt the diameter (32) on A, ſtands (100.6) the circumference on B.

PROB.

PROB. XVIII.

To find the diameter of a circle from the circumference.

1. RULE.

Say as 22 to 7, or as 355 to 113, or as 3.1416 to 1; ſo the circumference to the diameter. 23.

2. *Or thus.*

Multiply the circumference by .3183, the product is the diameter.

Examp.

Suppoſe the circumference be 50.

```
       50
        7
      ---
22)250(15.91 the circumference.
   22
   ---
    130
    110
    ---
     200
     198
     ---
       2
```

By the Rule.

Set 22 on A, to 7 on B, and againſt the circumference (50) on A, is the diameter (15.9) on B.

Fig.

PROB. XIX.

The baſe and height of the ſegment of a circle being given; to find the diameter.

RULE.

12. As height of the ſegment DF :
to half the chord or baſe AF or FB : :
So the ſame half :
to a fourth GF ;
then this fourth proportional GF, added to the height of the ſegment FD, gives the diameter GD.

Examp.

Suppoſe DF 2 feet; AF, 3. Then 2 : 3 :: 3 : $\frac{9}{2}$ = $4\frac{1}{2}$ = FG, then adding DF, we have GD $6\frac{1}{2}$.

PROB. XX.

Having the radius of a circle; to find the ſide of a regular poligon inſcribed in it.

RULE.

Multiply the given radius, by the multiplier, over againſt the number of ſides in this table; the product is the ſide of the poligon.

Numb. of ſides.	Multipliers.
3	1.732051
4	1.414214
5	1.175570
6	1.000000
7	0.867767
8	.765367
9	.684040
10	.618034
11	.563465
12	.517638

Ex. 1.

Suppoſe the radius of a circle 5, *what is the ſide of an inſcribed equilateral triangle?*

1.73205
5

8.66025 Anſ.

Fig.

Ex. 2.

If the radius of a circle be 315, *what is the ſide of an inſcribed octagon?*

$$\begin{array}{r} .76536 \\ 315 \\ \hline 382680 \\ 76536 \\ 229608 \\ \hline 231.08840 \end{array}$$ Anſwer.

PROB. XXI.

To find the length of the arch of a circle, ADB, *by having the chord* AB.

1 RULE

Divide the chord AB into four equal parts, and ſet one part from B to E, upon the arch; and from A to H upon the chord; and draw HE; then 2HE is the length of the arch ADB, the leſſer ſegment. 12.

2. *Otherwiſe.*

Make arch AD = DB, and draw the chord AD; then $\frac{8AD - AB}{3}$ = length of the arch ADB.

3. *Otherwiſe.*

Make AD = DB, and draw the diameter DFG. Then $\frac{DG - .16DF}{DG - .82DF} \times AB$ = length of the arch ADB.

Note, if the arch is greater than a ſemicircle; find the length of half of it by this rule, and double it.

Fig. 12. This Prop. may more eaſily be reſolved, by putting a ſtring round ADB, and then meaſuring the ſtring.

Examp.

Suppoſe AB *be* 7; DF, 2. Then by Prob. 18. DG = $8\frac{1}{8}$.

.16
2
.32=.16DF
.82
2
1.64=.82DF

8.125
.32
7.805
7
6.485)54635(8.42
51880
2751
2594
161

8.125
1.640
6.485

So the length of ADB is 8.42.

PROB. XXII.

To find the length of the quadrant of a circle, AB.

RULE.

13. Divide the ſemicircle AGC into three equal parts at G, H; which is done by ſetting CO from C to H, and from A to G; and draw BG, BH. Make DF = DE; then BF is equal to the quadrant BA or BC.

Or thus.

$1\frac{1}{9}$ the chord AB = quadrant AIB.

Examp.

Let the radius AO *be* 20;
Then by Prob. 14, AB is 4.24;

9)4.34(.47
36
64

4.24
.47
4.71 = quad. AIB.

PROB.

PROB. XXIII.

To find the periphery of an ellipſis.

1. RULE.

Multiply half the ſum of the tranſverſe and conjugate diameters, by 3.1416; the product is the periphery.

2. Or thus.

To twice the tranſverſe add $1\frac{1}{7}$ the conjugate for the periphery.

Examp.

Let the tranſverſe be 3, and the conjugate 2.

$$\begin{array}{r} \text{Then } 1\frac{1}{7}\times 2 = 2\frac{2}{7} = 2.285 \\ 6. \\ \hline \text{Anſ.}\quad 8.28 \end{array}$$

Otherwiſe.

$$\begin{array}{r} 3 \\ 2 \\ \hline 2)5(2\frac{1}{2} \end{array} \qquad \begin{array}{r} 3.1416 \\ 2\frac{1}{2} \\ \hline 62832 \\ 15708 \\ \hline \text{Anſ.}\quad 7.8540 \\ \hline \end{array}$$

PART III.

The Meaſuring of Areas and Surfaces.

PROB. I.

To find the area of a triangle ABC.

1 RULE.

14, MULTIPLY the baſe AB, by the perpendicular
15, CD, half the product is the area.

2. *Or thus.*

Take half the ſum of the three ſides, and ſubtract each ſide from it; then multiply that half ſum and theſe three remainders together; the ſquare root of the laſt product is the area.

3. *Or thus.*

Find the nat. ſine of the angle A, and multiply that ſine, and the ſides AC, and AB, together; half the laſt product is the area. Or elſe uſe logarithms.

If CD be not perpendicular, but drawn to make any angle you will at D. Then add the logarithms of AB, CD, and ſine <D, together; abate 10. Then find the number belonging; half of it is the content.

Fig.

Examp. 14.

The baſe AB *is* 24, *the perpendicular* CD, 13. 15.

13
12

26
13

156

By the Rule.

Set 1 on A, to half the baſe (12) on B; then againſt the perpendicular (13) on A, is the content (156) on B.

PROB. II.

To find the area of a ſquare ABCD.

RULE.

Multiply any ſide by itſelf; the product is the area. This is called ſquaring the ſide AB. 16.

Examp.

Suppoſe the ſide of the ſquare 23 *inches.*

23
23

69
46

Anſ. 529

By the Rule.

Set 1 on A, to the ſide 23 on B, then againſt 23 on A, is the area 529 on B.

PROB. III.

To find the area of a right angled parallelogram ABCD.

RULE.

17. Multiply the longer ſide by the ſhorter; the product is the area.

Examp.

Suppoſe the ſides 16 *and* 35.

```
 35
 16
---
210
35
---
560 area,
---
```

By the Rule.

Set 1 on A to the length (35) on B; and againſt the breadth on A (16), is the area (560) on B,

SCHOLIUM.

To find the number of tiles to cover a houſe.

Find the ſquare yards in the floor or baſe, which multiply by 23, if a ſquare roof. Or by 25 if ſharper; or by 32 if true pitch. Generally a quarter of the ſquare yards, ſhews how many hundred are required.

PROB. IV.

To find the area of a Rhomboides, or oblique Parallelogram, ABOD.

RULE.

18. Multiply the length DO or AB, by the breadth, or perpendicular AE; the product is the area.

Examp.

Fig.
18.

Examp.

The length AB 27 *feet, the breadth* AE 18.

27
18

216
27

486. area.

By the Rule.

Set 1 on A, to the length (27) on B; then againſt the breadth (18) on A, is the content (486) on B.

PROB. V.

To find the area of a Trapezoid ABCD.

RULE.

Multiply the ſum of the parallel ſides AB, DC, by the perpendicular diſtance between them AE; half the product is the area or content. 18.

Ex.

Suppoſe the length AB 30 *feet,* DC 37; *and the breadth* AE 18; *to find the area.*

30 87
57 18

87 696
87

2)1566.

783 area.

By

Fig.

By the Rule.

Set 1 on A, to half the ſum of the parallel ſides (43½) on B; and againſt the breadth (18) on A, is the area (783) on B.

PROB. VI.

To find the area of a Trapezium ABCD.

RULE.

19. 20. Meaſure the diagonal AC, and the perpendiculars thereon BE, DF. Then multiply the diagonal, by the ſum of theſe perpendiculars; half the product is the area.

Ex.

AC = 28

BE = 13
DF = 25

38
28

304
76

1064
Area, 532

By the Rule.

Set 1 on A to 28 on B, then againſt 19 (half of 38) on A, is the content (532) on B.

Fig.

PROB. VII.

To find the content of any irregular plain figure.

RULE.

Divide it into triangles and trapezia, by diagonal lines. And meaſure all theſe triangles and trapezia ſeparately, by the former Rules; and the ſum of them all is the area or content. 21.

If any ſide be crooked, draw a ſtreight line, that ſhall leave as much out, as it takes in.

Examp.

In the figure annext, there are eight ſides, and it is divided into two trapeziums, ABGH, and BDEC; and two triangles BCD, GEF. In the trapezium ABGH, AG = 12, BI = 4, HK = 4.25; In the trapezium BDEG, GD = 167, BM = 8.6, EN = 5.7. In the triangle BCD, BD = $10\frac{1}{2}$, CO = 4.71. In the triangle GEF, GE = 12,36, LF = 3.25.

BI = 4. BM = 8.6
HK = 4.25. EN = 5.7

8.25. 14.3, then by Logarithms,

$\frac{1}{2}$AG,

Fig. 21.

½AG,	6	0.77815	
Sum. per.	8.25	0.91645	
Area	49.5	1.69460	49.5
			119.4
½GD,	8.35	0.92168	24.72
Sum. per.	14.3	1.15533	20.08
Area	119.4,	2.07701	213.70
			whole area.
½BD	5.25,	0.72015	
CO	4.71	0.67302	
Area	24.72	1.39317	
½GE	6.18	0.79098	
LF	3.25	0.51188	
Area	20.08	1.30286	

PROB. VIII.

To find the area of a regular Poligon.

1. RULE.

22. Meaſure the perpendicular diſtance CP from the center of the figure to one ſide AG.

Then multiply the perpendicular CP by the circumference of the poligon ABDEFG; half the product is the area.

2. *Other-*

2. *Otherwiſe.*

Multiply the given ſide of the poligon by itſelf, and then by the multiplier over againſt the number of ſides, in this Table; the product is the area.

Fig. 22.

number of ſides.	Multipliers.
3	0.433013
4	1.000000
5	1.720477
6	2.598076
7	3.633912
8	4.828427
9	6.181824
10	7.694209
11	9.365640
12	11.196152

Examp.

Let the ſide of a hexagon be 12 inches.

12 2.598
12 144
—— ——
24 10392
12 10392
—— 2598
144 ——
—— area, 374.112

By the Rule.

Set 1 on A to 144 on B, and againſt 2.6 on A is the area 374 on B.

PROB. IX.

To find the area of a circle ABCD.

1. RULE.

Multiply half the circumference ABCD, by half the diameter AC; and the product is the area. 23.

2. *Or thus.*

Make as 14 to 11; ſo the ſquare of the diameter AC, to the area.

3. *Or*

Fig. 23.

3. *Or thus.*

Multiply the ſquare of the diameter by .7854, the product is the area.

4. *Or thus.*

Say as 88 to 7, ſo the ſquare of the circumference, to the area.

5. *Or thus.*

Multiply the ſquare of the circumference by .07958; the product is the area.

You may uſe any one of theſe rules, according as the data happens to be.

Ex. 1.

Let the diameter be 26, *to find the area.*

26	.7854
26	676
156	47124
52	5498
	471
676	
	530.93 area.

1. *By the Rule.*

Set 14 on A, to 11 on B; then againſt the ſquare of the diameter on A (676), is the area on B (531).

2. *Or thus by the Rule.*

Set 1 on D, to .785 on C;	then againſt the diameter (26) on D, is the area (531) on C.
or ſet 1.13 on D, to 1 on C;	
or ſet 11 on D, to 95 on C;	
or ſet 8 on D, to 50 on C;	
or ſet 12 on D, to 113 on C;	

Fig. 23.

Ex. 2.

Suppoſe the circumference be 5, what is the area?

```
 5        .0796
 5           25
---      -------
25         3980
          1592
         -------
         1.9900, or 2 the area.
```

3. *By the Rule.*

Set 88 on A, to 7 on B, and againſt the ſquare of the circumference (25) on A, is the area (1.99) on B.

4. *Or thus by the Rule.*

Set 10 on D, to 7.96 on C;
or ſet 11.3 on D, to 10 on C;
or ſet 5 on D, to 2 on C;
or ſet 11 on D, to 9.6 on C;
then againſt the circumference (5) on D, is the area (1.99) on C.

Cor. *To meaſure the area of a ring or annulus.*

1. RULE.

Find the areas of the outer and inner circles, and their difference is the area of the annulus.

2. *Or thus.*

Multiply the ſum of the diameters by their difference, and the product by .7854.

SCHOL.

The reaſon of theſe different proceſſes is founded on this, that the diameter is to the circumference, as 1 to 3.1416, or nearly as 7 to 22; and the ſquare of the diameter to the area, as 1 to

3.1416

Fig. 23. $\frac{3.1416}{4}$ or .7854; that is nearly as 14 to 11. Likewiſe the ſquare of the circumference, to the area; is as 1 to .07958, or as 22 × 22 to $\frac{7 \times 22}{4}$, or as 22 × 4 to 7, or 88 to 7. And upon the rule, the numbers on C are as the ſquares of the numbers on D. Therefore it will be, $1^2 : .785 :: \overline{1.13}^2 : 1 :: 11^2 : 95 :: 8^2 : 50$, which are all in the ſame proportion. For the ſame reaſon, $1^2 : .0796 :: \overline{11.3}^2 : 10 :: 5^2 : 2 :: 11^2 : 9.6$, are in the ſame proportion, nearly.

Likewiſe in the ſquare which is equal to the circle, whoſe diameter is 1, the ſide is $.886227 = \sqrt{.7854}$. And the ſide of the inſcribed ſquare is $.707107 = \sqrt{\frac{1}{2}}$.

Alſo in the circle whoſe circumference is 1, the diameter is $.318310 = \frac{1}{3.1416}$; and the area $.079577 = \frac{.7854}{3.1416}$; and the ſide of the equal ſquare, is $.282095 = \frac{.886227}{3.1416}$. And ſide of the inſcribed ſquare is $.225079 = \frac{707107}{3.1416}$.

Whence as 7 to 11, ſo the inſcribed ſquare to the circle's area.

PROB. X.

To find the area of the ſector of a circle ADC.

RULE.

24. Multiply the arch AD by the radius AC, and half the product is the area.

Ex.

Fig. 24.

Ex.

Let the radius AG *be* 10, *and the arch* AD 7.

$$\begin{array}{r} 10 \\ 7 \\ \hline \end{array}$$

2)70(35 area.

PROB. XI.

To find the area of the ſegment of a circle, ADB.

1 RULE.

Multiply the baſe AB by the height DE; and $\frac{2}{3}$ the product is the area, when the height is ſmall; or $\frac{3}{4}$ the product, if near a ſemicircle.

2. *Otherwiſe.*

Let DE be the height, and AE or EB half the baſe; then multiply the ſquare of DE, by .392, to which add the ſquare of AE. Extract the ſquare root of the ſum, which multiply by $\frac{4}{3}$ DE, for the area.

3. *Otherwiſe.*

Let D be the vertex, and draw the chord AD; to the ſquare of AB, add the ſquare of DE, extract the ſquare root of the ſum; to twice that ſquare root add AD, which multiply by $\frac{4}{15}$ DE for the area.

4. *Otherwiſe.*

Find d the diameter, then $\frac{80d - 39\mathrm{DE}}{16d - 3\mathrm{DE}} \times \mathrm{AD} \times \frac{4}{15}\mathrm{DE}$, is equal to the area.

Fig. 24.

5. *Otherwiſe.*

Find d the diameter, then $\frac{5d - 3DE}{5d - 4DE} \times \frac{2}{3}AB \times DE$ = the area of the ſegment.

Note, a zone adjoining to the center may be found by this Rule, by ſubtracting the ſegment from the ſemicircle.

6. *Otherwiſe.*

$\frac{4AD + 3AB}{5} \times \frac{2}{3}DE$ = area of the ſegment ADB.

Note, the 1, and 5th Rules, will alſo find the area of an elliptic ſegment.

7. *Or thus.*

Let DI = IE, then $\frac{AD + 4AI}{3} \times \frac{4}{5}DE$ = area of the ſegment.

8. *Or thus.*

Let arch AD + AE = p, arch AD — AE = q. Then $\frac{p \times DE^2 + q \times AE^2}{2DE}$ = area of the ſegment.

Examp.

Let the baſe AB *be* 32, *chord* AD 20, *height* DE 12. *Then by the 6th way,*

20	32	12
4	3	2
80	96	3)24(8
	80	
		35.2
	5)176(35.2	8
		281.6 the area.

When the ſegment is greater than a ſemicircle, find the leſſer ſegment, which ſubtract from the whole circle.

PROB.

PROB. XII.

To find the area of an ellipſis ACBD.

RULE.

Multiply the tranſverſe AB by the conjugate axis CD; and the product by .7854, for the area. 8.

For an elliptic ſegment, ſee the laſt Problem.

Example.

Suppoſe the tranſverſe be 32, *and the conjugate* 24.

```
 32        .7854
 24          768
----      -------
128        62832
64         47124
----      54978
 768      --------
          603.1872  area.
          --------
```

By the Rule.

Find a mean 27.7 between 32 and 24. Then ſet 1 on D to .785 on C; and againſt 27.7 on D, is the content 603. on C.

PROB. XIII.

To find the area of a Parabola ABC.

RULE.

A parabola is made by cutting a cone by a plane parallel to its ſide. 25.

Multiply the baſe AC by the height BD; and $\frac{2}{3}$ that product is the area.

Fig. 25.

Examp.

Suppoſe the height BD = 100, *and baſe* AC = 68.

68	6800
100	2
6800	3)13600(4533, the area.

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (33.3) on B, and againſt twice the baſe (136) on A, is the content (4533) on B.

PROB. XIV.

To find the ſegment AEFC *of a Parabola.*

RULE.

EF is parallel to AC, then $\frac{AC^3 - EF^3}{AC^2 - EF^2} \times \frac{2}{3}GD$ = the area.

Examp.

Let AC *be* 12, EF 8, GD 5, *to find the area.*

12	8	144	80)1216(15.2	15.2
12	8	64	80	5
144	64	80	416	76.0
12	8		400	2
		1728		
1728	512	512	160	3)152.0
			160	50.66 area.
		1216		
			0	

PROB.

Fig. 25.

PROB. XV.

To meaſure irregular Parallelograms, ſuch as irregular planks, &c.

RULE.

Take the breadth in ſeveral equidiſtant places, the ſum of which divide by the number of places, for the mean breadth; multiply the mean breadth by the length, for the area.

Examp.

Suppoſe a plank 16 *feet long, and the breadth in ſeveral places* 6, 7, 9, 9, 11 *inches.*

6	16
7	8.4
9	——
9	64
11	128
——	——
5)42(8.4 mean breadth.	12)134.4(11.2 feet.

By the Rule.

Having found the mean breadth in inches; then ſet 12 on A, to the length 16 on B; and againſt the breadth 8.4 on A, is the content on B 11.2 feet.

PROB. XVI.

To meaſure any irregular figure, bounded by a curve.

1. RULE.

Draw AC for the baſe, then meaſure the three equidiſtant perpendiculars Aa, Bb, Cc; then 26.

$$\frac{Aa + Cc + 4Bb}{6} \times AC = \text{area } AacC.$$

2. Otherwiſe.

27. Meaſure the four equidiſtant perpendiculars A*a*, B*b*, C*c*, D*d*; then

$$\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD = \text{area } Aa\,dD.$$

3. Or thus more exactly.

28. Meaſure the five equidiſtant perpendiculars A*a*, *Bb*, C*c*, D*d*, E*e*; then

$$\frac{7Aa + 7Ee + 32Bb + 32Dd + 12Cc}{90} \times AE = \text{area}$$

A*ae*E; but this is troubleſome to complete.

29. And the ſame holds, if the figure be curve on both ſides; drawing the baſe AC thro' it.

4. Otherwiſe.

30. Divide the baſe AI into any even number of equal parts, and

Let X equal ſum of the extreme ordinates A, I.
O = ſum of all the other odd ones, the 3d, 5th, 7th, &c.
E = ſum of all the even ones, 2d, 4th, 6th, &c.
L = length of two of theſe equal parts. Then

$$\frac{X + 2O + 4E}{6} \times L = \text{area of the curve.}$$

Where for every two ordinates, from the firſt; the curve ought to be concave the ſame way.

Examp.

26. *There is a curve line whoſe baſe is* 67 *feet, and three ordinates or perpendiculars erected*, A*a*, B*b*, C*c*, *are* 35, 40 *and* 28.

By the firſt Rule.

```
                 35     223
                 28      67
   40           160     ----
    4           ---    1561
   ---          223    1338
   160          ---    ----- area.
                  6)14941(2490 1/6.
```

PROB.

PROB. XVII.

To find the ſurface of a vaulted roof, being like the arch of a bridge.

RULE.

Meaſure the length of the curve of the arch by a ſtring or otherwiſe; then multiply this length of the arch, by the length of the vault; the product is the ſurface.

Examp.

Suppoſe the arch be 30 *yards, and the length of the vault* $7\frac{1}{2}$.

$$\begin{array}{r} 30\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 210 \\ 15 \\ \hline \end{array}$$

Surf. 225 yards.

Cor. *If the arch be a ſemicircle, or any ſegment of a circle; its length may be found by* Prob. II. *without meaſuring.*

PROB. XVIII.

To find the ſurface of a pyramid, parallelopipedon, or of any ſolid bounded by planes.

RULE.

Meaſure every particular plane ſeparately; and the ſum of all is the ſurface.

Cor. *If it be an upright priſm; meaſure the circumference of the baſe, which multiply by the height of the ſolid.*

Fig.

PROB. XIX.

To meaſure the ſurface of a right cone, or a fruſtum of it.

RULE.

31. Multiply the circumference of the baſe by half the ſlant ſide, for the ſurface of the whole.

Or multiply half the ſum of the circumferences of the top and bottom, by the ſlant ſide, for the fruſtum.

Or thus.

If AB be the ſide, and BC the Diameter. Then as 7 : 11 : : AB × BC : ſurface of the cone.

Example.

Let the ſide AB *be* 20; *diameter* BC 8.

20
8

160
11

160
160

7) 1760
251.4
Surface.

PROB. XX.

To find the ſurface of a right cylinder.

1. RULE.

32. Multiply the circumference of the baſe AB by the height AC, for the convex ſurface.

2. *Or thus.*

As 7 : 22 : : AB × AC : ſurface.

Examp.

Fig.

Examp.

Let the ſide CA *be* 20, *diameter* AB 8. 32.

20
8
———
160
22
———
320
320
——— ſurface.
7) 3520(502.8
———

PROB. XXI.

To find the ſurface of a ſphere.

1. RULE.

Multiply the circumference of the ſphere ABCD, by the diameter AC, for the ſurface.

2. *Or thus.*

Find the area of a great circle of the ſphere, which multiply by 4 for the ſurface. 33.

Or thus.

7 : 22 : : AC² : ſurface.

Examp.

Let AC = 9 *feet.*

9 81
9 22
— —
81 162
162
———
7)1782
254,6 ſurface.

Examp.

PROB. XXII.

To find the ſurface of the ſegment of a ſphere.

1. RULE.

34. Multiply the circumference of the whole ſphere by the height of the ſegment CD, for its ſurface.

2. *Or thus.*

To the ſquare of the diameter of the baſe add 4 times the ſquare of the height CD; the ſum multiplied by .7854, gives the ſurface; or $3.1416AC^2$ = ſurface.

3. *Or thus.*

As 7 to 22, ſo AC^2 to the ſurface ACB.

Cor. *Hence if the ſurface of any zone be required, work by the firſt rule,* viz. *multiply the circumference of the whole ſphere, by the height of the zone.*

Examp.

If AC *be* 25 *feet, what is the ſurface* ACB?

$$\begin{array}{r}25\\25\\\hline125\\50\\\hline625\end{array}\qquad\begin{array}{r}625\\22\\\hline1250\\1250\\\hline7)13750(\ 1964.3\text{ feet.}\end{array}$$

PROB.

Fig. 35.

PROB. XXIII.

To find the ſurface of a cylindric ring.

1. RULE.

Add the thickneſs AB to the inner diameter BD, which ſum multiply by the thickneſs AB, and the product by 3.1416^2, gives the ſurface.

2. *Or rather thus.*

The ſurface $= AB \times AD \times 9.87$.

3. *Or thus.*

From $10AB \times AD$, ſubtract $\frac{1}{77}$ part thereof, for the ſurface.

Examp.

Let AB *be* 1, AD 12.

12	9.87
1	12
12	1974
	987
	118.44 ſurface.

PROB. XXIV.

To find the ſurface of a ſpheroid.

1. RULE.

Multiply the two axes AB CD together, and that product by 3.1416 for the ſurface of an oblong ſpheroid. 36.

2. *Or*

Fig. 36.

2. *Or thus.*

To twice the tranſverſe (or axis of revolution) add the conjugate diameter, multiply the ſum by $\frac{1}{3}$ the conjugate; and the product by 3.1416 gives the ſurface.

3. *Otherwiſe.*

Meaſure the circumferences about AB and CD, which call P and Q. Then as 22 : 7 : : P × Q : ſurface of the ſpheroid; which is a ſmall matter too big, when the diameters AB, CD, are very unequal.

Examp.

Let AD *be* 10, CD 7.

10	3.1416
7	70
70	219.9120 or 220 the ſurface.

PROB. XXV.

To find the ſurface of any ſolid CAD, *generated by revolving about the axis* AB.

RULE.

37. Take with a ſtring the length of the arch CA or AD; and divide the ſtring into 3 equal parts at *e*, *f*; apply the ſtring again to the curve, and mark where the points *e*, *f*, fall: take the diameters there, which call P and Q, then $\frac{CD + 3P + 3Q}{8} \times 3.1416\ AfeC =$ ſurface.

Note, If the curve does not come to the axis at A, you muſt take its diameter there, and add it to CD.

If the body be very long, divide it into two or more parts, and meaſure each part ſeparately, by this Rule. Fig. 37.

Examp.

Suppoſe the three diameters at C, *e and f*; *that is* CD, P *and* Q, *to be* 12, 10, 7; *and the length of the curve* A*fe*C, 15.

10	7	12	3.1416	
3	3	30	63	
—,	—	21	—	197.92
30	21	—	94248	15
—	—	63	188496	—
		—	—	98960
			197.9208	19792
			—	—
			8)	2968.80
				371.10
				Surface.

Schol.

Any proportion on the lines A and B on the ſliding rule, may be wrought on Gunter's line, with a pair of compaſſes; by extending from the firſt term to the ſecond; and ſetting that extent the ſame way, from the third to the fourth.

Or, extending from the firſt to the third; and ſetting that extent from the ſecond to the fourth.

Alſo, any of theſe proportions, which are wrought on the lines D and C, may be wrought on Gunter's line with compaſſes, thus. Extend from the firſt term to the third; and ſet twice that extent (the ſame way) from the ſecond term, and it will reach to the fourth.

Examp. If the diameter of a circle be 26 (*Ex.* 1. Prob. IX,), extend from 11 to 26, and that extent

Fig. 37. tent (turned twice over) will reach from 95 to the area 531.

And the ſame rule holds good for ſolids, by the proportions laid down in the following parts of this book.

Alſo if the diameter or ſide of a ſolid is known, and its content; the content of any other ſimilar ſolid may be found; whoſe diameter or ſide is given. Thus

Extend from the known ſide to the given ſide, and thrice that extent will reach from the known content to the content required.

PART IV.

The Meaſuring of Solids.

PROB. I.

To find the ſolidity of a Cube ABCD.

RULE.

MULTIPLY the ſide AB by itſelf, and that product by the ſame ſide again; and the laſt product is the ſolid content, in the ſame ſort of meaſure. 38.

This is called *cubing* the ſide AB.

Examp.

Suppoſe the ſide AB, *be two feet two inches.*

2 f. 2 in. = 2.166 feet.

```
      2.166           4.691
                      2.166
     ———————        ———————
      12996
     12996            28146
     2166            28146
    4332             4691
    ————————        9382
   4.691556         ———————
    ————————        10.160706
               or   10.16 feet.
                    ———————————
```

1. *By*

Fig. 38.

1. *By the Rule.*

Set 1 upon D, to the ſide (2.166) on C; and againſt the ſide (2.166) on D, is the content on C (10.16) feet, being the ſame ſort of meaſure.

2. *Or thus, by the Rule.*

Set 12 upon D, to the height in feet (2.16) on C; and againſt the ſide in inches (26) on D, is the content in feet (10.16) on C.

PROB. II.

To find the content of a ſquare priſm, or parallelopipedon with a ſquare baſe, ABCD.

RULE.

40. Square the ſide AB for the area of the baſe; which multiply by the height, gives the ſolidity.

Examp.

Suppoſe the baſe 2 *feet* 6 *inches, and the height* 19 *feet.*

2 f. 6 in. =	2.5	6.25
	2.5	19
	125	5625
	50	625
baſe	6.25	118.75 content.

1. *By the Rule.*

Set 1 upon D, to the height (19) upon C, and againſt the ſide of the baſe (2½) upon D, is the content (118.7) upon C, in the ſame meaſures.

2. *Or thus, by the Rule.*

Set 12 upon D, to the height in feet (19) upon C, and againſt the ſide of the baſe in inches (30) upon D, is the content in feet (118.7) upon C.

PROB. III.

To find the ſolidity of any ſort of priſm, ABCD.

RULE.

Find the area of the baſe AB, by ſome of the 41.
Problems in Part III. then multiply that baſe by 42.
the height AC, gives the ſolid content.

Exam.

Let the baſe AB, *be* 4.65 *feet, and height* AC 25 *feet.*

```
 4.65
   25
 ----
 2325
 930
 ------
 116.25 feet.
```

1. *By the Rule.*

Set 1 upon A, to the baſe (4.65) upon B; then againſt the length (25) on A, is the content on B (116), in the ſame ſort of meaſure.

2. *Or thus, by the Rule.*

Set 144 upon A, to the baſe in inches ($669\frac{1}{2}$) on B, then againſt the length (25) in feet on A, is the content in feet on B (116).

Fig.

PROB. IV.

To find the ſolid content of a cylinder, ABCD.

RULE.

39. Find the area of the baſe AB or DC, by Prob. 9. Part III. then multiply that baſe by the height AD, for the content.

Examp. 1.

Let the diameter of a cylinder be 14 *inches, and the height* 5 *feet.*

14 in. = 1.166 +	.785	1.067
1.167	1.36	5
8162	4710	5.335 content.
6996	2355	
1166	785	
1166		
	1.06760 baſe.	
1.360722		

1. *By the Rule.*

Set 1.13 upon D, to the height (5) upon C, and againſt the diameter (1.166) on D, is the content (5.33) on C, in the ſame meaſure.

2. *Or thus, by the Rule.*

Take ¼ the circumference for the girt. Then ſet .886 on D, to the height (5) on C; then againſt the girt (.917) on D, is the content (5.33) on C; in the ſame meaſure.

For here the diameter being 14 inches the circumference is 44, and the girt 11 or .917 feet.

Fig. 39.

3. *Or thus, by the Rule.*

Set 3.54 upon D, to the height (5) on C; and againſt the circumference (3.66) on D, is the content (5.33) on C; in the ſame meaſure.

Ex. 2.

There is a cylinder whoſe height is 40 *feet, and diameter* 30 *inches, or circumference* 94.25 *inches; to find the content in feet.*

30 in. = 2.5 f.	.7854	4.908
2.5	6.25	40
125	47124	196.320 content.
50	1571	
	392	
6.25		
	4.9087	

4. *By the Rule.*

Set 13.54 on D, to the height in feet (40) on C; and againſt the diameter in inches (30) on D, is the content in feet (196.3) on C.

5. *Or thus, by the Rule.*

Take $\frac{1}{4}$ the circumference in inches for the girt. Then ſet 10.63 on D, to the height in feet (40) on C; and againſt the girt in inches (23.5) on D, is the content in feet (196.3) on C.

6. *Or thus, by the Rule.*

Set 42.54 on D, to the height in feet (40) on C; and againſt the circumference in inches (94.25) on D, is the content in feet (196.3) on C.

Fig.
39.

SCHOLIUM.

Obſerve here, that .886227 is the ſquare root of the area of a circle whoſe diameter is 1 $= \sqrt{.7854}$.

And .282095 is the ſide of a ſquare, equal to a circle, whoſe circumference is 1.

And $3.5450 = \frac{1}{.282095} = \frac{3.1416}{.8862}$, and is the circumference of a circle, whoſe area is 1.

And 10.63472 is the ſide of a ſquare, equal to a circle, whoſe diameter is 12 inches $= \sqrt{.7854 \times 144} = 12 \times .886227$.

And $42.538896 = 4 \times 10.634724 = 12 \times 3.5450$.

And $13.54055 = \frac{42.538896}{3.1416}$, and is the diameter of a circle, whoſe area is 144, and therefore is $= \frac{12}{.8862}$.

And 1.128381 is the diameter of a circle, whoſe area is 1, $= \frac{1}{.8862}$.

PROB. V.

To find the content of a cone or pyramid ADB.

RULE.

43. 44. Meaſure the baſe, then multiply the baſe by one third of the height DC; or elſe multiply the baſe by the height, and take $\frac{1}{3}$ the product, for the content.

Examp.

Fig.

Examp. 43.

There is a cone or pyramid whoſe height DC *is* 60, 44.
and baſe 51, *to find the content.*

51 or	51
20	60
1020 content	3)3060
	1020 content.

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (20) on B; and againſt the baſe (51) on A, is the content (1020) on B.

PROB. VI.

To find the content of the fruſtum of a cone or pyramid AD.

1. RULE

Find the areas of the two baſes, at top and bottom AB, CD, and the height PQ; call the baſes 45.
B and *b*. Then $\frac{B+b+\sqrt{Bb}}{3} \times PQ =$ content. 46.
That is, multiply the two baſes together, and extract the ſquare root; to the ſum of the baſes add this ſquare root, and multiply the third part thereof by the height.

2. *Or thus.*

For a ſquare pyramid, to the rectangle of the ſides of the two baſes, add $\frac{1}{3}$ the ſquare of the difference; the ſum multiplied by the height, gives the ſolidity of the fruſtum.

In a cone uſe the diameters inſtead of the ſides; and the laſt product muſt be multiplied by .7854, for the fruſtum.

3. *Other-*

Fig.
45.
3. *Otherwiſe.*

46. In a ſquare pyramid, let D, *d*, be the ſides of the baſes, or diameters in a cone. Then $\frac{D^3 - d^3}{D - d} \times \frac{1}{3}$ the height, = content of the ſquare fruſtum; which fruſtum muſt be multiplied by .7854, for the content of the conic fruſtum.

Examp.

Suppoſe the greater diameter 36, *the leſſer* 27, *and the height* 24.

```
 36      36      972     23976 the ſquare fruſtum,
 27      27       27     .7854
---      --      ---     -----
252       9      999    167832
 72       9       24     19181
---      --      ---      1198
972   3)81(27   3996        95
---    ----     1998     -----
                ----   18830.6 the
               23976   conic fruſtum.
               -----   -------------
```

P R O B. VII.

To find the content of a Priſmoid or Pyramidoid GE.

47. This is a ſolid whoſe ſides are ſtreight, and the two baſes parallel, but not ſimilar figures; when they are poligons, 'tis called a *Priſmoid*, or *Pyramidoid*; but if they are ellipſes, 'tis called a *Cylindroid*. For this ſolid, when the baſes are rectangles, or ellipſes.

R U L E.

To the bottom length (BE), add half the top length (AD); and multiply the ſum by the bottom breadth (BC), for the firſt product.

To the top length (AD), add half the bottom length (BE), and multiply the ſum by the top breadth (AG); for the ſecond product. Fig. 47.

Multiply the ſum of theſe products by $\frac{1}{3}$ the height (AL) for the content.

When the body is a cylindroid, with elliptic baſes; you muſt uſe the diameters inſtead of the ſides parallel to them; and compute the ſolidity as before; and then multiply the laſt product by .7854, and it gives the content of the inſcribed cylindroid.

When the baſes are poligons, you may uſe the firſt rule in the laſt Problem.

Examp.

There is a ſquare priſmoid, whoſe dimenſions are BE = 40, AD = 36, CB = 32, AG = 12; *and height* AL = 28.

40	36	1856
18	20	672
58	56	2528
32	12	$9\frac{1}{3}$
116	112	22752
174	56	$842\frac{2}{3}$
1856	672	$23594\frac{2}{3}$ content.

But for the inſcribed cylindroid, multiply by .7854.

23594.6
.7854
1651626
188757
11797
943
Content, 18531.23

Fig. 48. 49.

PROB. VIII.

To find the content of any ungula, or hoof of a cone.

This is a ſolid made by cutting the fruſtum of a cone, by a plane paſſing through it diagonally, or touching the two oppoſite baſes, on contrary ſides.

1. RULE.

Meaſure the diameter of the baſe AB, and the conjugate diameter IO, of the ellipſis AICO; alſo the height CD. Then divide the difference of the cubes of AB, IO, by the difference of their ſquares; and multiply the quotient by $.7854 AB \times \frac{1}{3}$ CD, for the content of either ungula.

Note, if d be $=$ the diameter that is wanting; then you may take $\sqrt{AB \times d}$ inſtead of IO.

2. *Or thus.*

Let diameter AB $= b$, conjug. diameter IO $= c$. Then $\frac{bb + bc + cc}{b + c} b \times \frac{.7854}{3} \times AB$, is the content.

Examp

Fig. 48. 49.

Examp.

Suppoſe AB 16, IO 9, CD 12; *to find the content.*

```
    16     9     16          481
    16     9      9           16
    ——    ——     ——          ————
    96    81    144         2886
   16     ——     81         .481
   ——           256          ———
   256           ——    25)7696(307.8
   ———          481         75..
                 ——          ——
   307.8                     196
      12       1231.2        175
   ————         .7854        ———
   6156        ——————         21
  3078          86184        ———
  —————          9849
3)3693.6          615
  1231.2           49
  ——————       ——————
               966.97 the content.
               ——————
```

PROB. IX.

To meaſure irregular parallelopipedons, ſuch as pieces of timber, &c.

1. RULE.

Girt it about in the middle with a ſtring, and meaſure it, and take a quarter of the girt in inches for the ſide of the ſquare; therefore ſquare it, and then multiply it by the length of the body; the product is the content.

2. *Otherwiſe.*

If the body is very irregular, divide it into two or more parts, and meaſure each part ſeparately, by this rule.

Or

Fig. Or elſe, gird it in ſeveral equidiſtant places, and add all the girths together, and divide the ſum by the number of places, for the girt; then proceed as before.

Note, if the girt be taken in inches, and the length in feet, the laſt product, divided by 144, gives the content in feet.

Examp.

A piece of wood is 18 *feet long, and the girt round about the middle, is* 66 *inches; to find the content in feet.*

A quarter of the girt is $16\frac{1}{2}$, for the ſide of the ſquare.

```
    16.5           272.25
    16.5               18
   ------          ------
     825           217800
    990            27225
   165             ------
   ------    144)4900.50(34.02 feet.
   272.25          432...
                   ----
                    580
                    576
                   ----
                    400
                   ----
```

3. *Otherwiſe by a Table.*

From this table, take the area correſponding to the ſide of the ſquare in inches; and multiply it by the length in feet for the content. If the ſide of the ſquare exceed the table, take half of it; and at laſt, take four times the content found.

A Table

A Table for meaſuring Wood.

Side ſquare	Area.	Side ſquare	Area.	Side ſquare	Area.
Inch.	Feet.	Inch.	Feet.	Inch.	Feet.
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2.506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1.361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
8½	.501	14½	1.460	23	3.673
8¾	.531	14¾	1.511	23½	3.835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5.062
10¾	.803	16¾	1.948	27½	5.252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.187	29½	6.044
				30	6.250

Examp.

If a piece of wood be 18 *feet long, and* 16½ *inches a quarter of the girt, as before.*

```
1.890
   18
-----
15120
1890
-----
34.020 the content as before.
```

1. *By the Sliding Rule.*

Meaſure the circumference in the middle, and take a quarter of it in inches, and call it the girt.

Then ſet 12 on D, to the length in feet (18) on C; then againſt the girt in inches (16½) on D, is the content on C (34) feet.

2. *Or thus, by the Rule.*

Take a quarter of the circumference, or the girt in feet, which here is 1.375; then,

Set 1 upon D, to the length (18) upon C; and againſt the girt (1.37) in feet on D, is the content (34) upon C.

Note, theſe are the ſame rules, as given in Prob. II.

SCHOL.

This is the common method of meaſuring wood, as practiſed in all raff yards, taking a quarter of the circumference in the middle, for the ſide of the ſquare. But it is objected, that it makes the content too little in proportion as 14 to 11; and that it ought to be meaſured as a cylinder. But to this it is anſwered, that before the wood can be ſquared and made fit for uſe, a great part of it goes to waſte in chips; and therefore the quantity of round timber ought to be reckoned no more than what the inſcribed ſquare will amount to, which ſhould be reduced in that proportion, that is as 11 to 7, which is more than 14 to 7. Another objection is, that in tapering timber, taking the ſide of the ſquare in the middle of the piece, makes the content alſo too little. But to this it is anſwered, that in moſt caſes, the great end is to be cut away, till it be of the ſame dimenſions as the leſſer end; or elſe it cannot be ſawn into proper ſtuff; and upon that account taking the ſquare in the middle makes too much of it. Fig.

On the other hand, when ſquare timber is meaſured, if one ſide of the ſquare is greater than the other, a quarter of the girt will make too much of it; ſo that this Rule will in ſome caſes give too much, and in others too little, by an inconſiderable quantity; and therefore may paſs well enough in common uſe. But if a round piece of wood is to be meaſured, every inch of which may be made uſe of, then I think it ought to be meaſured as a cylinder, or elſe as the fruſtum of a cone, if it be tapering, according to the Rules in Prob. 4th and 6th.

PROB. X.

To meaſure a heap of ſand, or earth; a pond of water, &c.

RULE.

Take the depth of it, in ſeveral places, at equal diſtances, as near as you can; add all theſe depths together, and divide by the number of them; the quotient is the mean depth. Then find the area of the baſe on which it ſtands, which multiply by the mean depth, and the product is the content.

If it happen, that the heap is ſo ſolid, that the depth cannot be taken, draw a ſtring over the top of it, parallel to the baſe, and meaſure the height

Fig. height of it above the baſe, for the greateſt depth of the heap. Then take ſeveral equidiſtant places upon the ſtring, and meaſure from the ſtring plumb down to the heap; then theſe being ſubtracted from the greateſt depth, give the depth in theſe ſeveral places. Then put the ſtring into a new poſition (or elſe draw another ſtring acroſs it), and take equal diſtances, and meaſure as before; and repeat the ſame, as oft as need be; till you get a ſufficient number of depths thro' the whole heap; all which muſt be ſet down and ſubtracted from the greateſt depth as before. And then you may apply the rule at firſt given.

Note, you may ſometimes reduce the heap into ſome ſort of a regular body, which will facilitate the meaſuring.

Examp.

There is a heap of ſtones, whoſe baſe is 15 yards; the greateſt depth 3, and the depth from lines drawn over it, taken in 9 places, are as in the table.

0	3.
$2\frac{1}{2}$	0.5
2	1
1	2
$1\frac{1}{2}$	1.5
2	1
1	2
$2\frac{1}{2}$	0.5
$1\frac{1}{2}$	1.5
	13.0

```
9)13(1.44            1.44
  9                    15
  —                   ———
  40                  720
  36                 144
  —                  ———
  4          content, 21.60 yards.
```

PROB.

PROB. XI.

To find the ſolidity of any of the five regular ſolids, from a ſide being given.

1. RULE.

Multiply the cube of the given ſide, by the number ſtanding againſt the name of the body in this Table; the product is the ſolid content.

Tetraedron or Pyramid.	.1178511
Hexedron, or Cube.	1.000000
Octaedron.	.4714045
Dodecaedron.	7.6631188
Icoſaedron.	2.1816951

2. *Or thus.*

Multiply the ſurface of the body by $\frac{1}{6}$ of the diſtance of two oppoſite faces, when they are parallel.

Examp.

The ſide of an Octaedron is 27.

$$\begin{array}{r} .4714 \\ 27 \\ \hline 32998 \\ 9428 \\ \hline 12.7278 \text{ content.} \\ \hline \end{array}$$

PROB.

PROB. XII.

To find the content of any irregular body bounded by planes.

RULE.

Suppoſe it divided into priſms and pyramids; then meaſure all theſe ſeparately, and the ſum of all is the content.

Examp.

50. The ſolid AHFD is divided into five parts, by planes drawn parallel to the ſides. There is 1. The parallelopipedon KHGLNFIO. 2. The triangular priſm AHKOIB. 3. The triangular priſm EFNOIC. 4. The ſquare pyramid IBDCO. 5. The priſm GPF. And each of theſe may be computed by the foregoing Problems, and the ſum will be the whole content. Part of this ſolid may be meaſured by Prob. VII. But when they are compounded they cannot, but muſt be cut in pieces, like this.

PROB. XIII.

To find the content of a cylindric ring.

RULE.

35. To the thickneſs of the ring, AB, add the inner diameter BD; multiply the ſum by the area of the ſection AB, and the product by 3.1416 gives the ſolidity.

Examp. Fig.

Let AB *the thickneſs be* 13, BD *the inner diameter* 157, *to find the ſolid content* BAED. 35.

157
13

170

13
13

39
13

169

.7854
169

70686
47124
7854

area, 132.7326
170

92912820
1327326

22564.54|20
3.1416

6769362
225645
90258
2256
1353

Content, 70888.74

PROB. XIV.

To find the ſolidity of a ſphere or globe, ABCD.

1. RULE.

Multiply its ſurface by the radius AO; $\frac{1}{3}$ the product is the ſolidity. 33.

2. *Otherwiſe.*

Say as 21 to 11, ſo the cube of the diameter, to the content: which is nearly as 2 to 1.

Fig. 33.

3. *Otherwiſe.*

Multiply the cube of the diameter by .5236; the product is the ſolidity.

4. *Or thus.*

Multiply the cube of the circumference by .0169; and you have the content.

5. *Or thus.*

Find the area of a great circle of the ſphere, and multiply it by $\frac{2}{3}$ of the diameter.

Examp.

Suppoſe the diameter of a globe is 16 *inches, or circumference* 50.3.

16	256	4096	21)45056($2145\frac{1}{2}$ content.
16	16	11	42..
96	1536	4096	30
16	256	4096	21
256	4096	45056	95
			84
			106
			105
			11

1. *By the Rule.*

Set 1.38 on D, to the diameter (16) on C, and againſt the diameter (16) on D, is the content (2140) on C, in the ſame meaſure.

Fig. 33.

2. *Or thus, by the Rule.*

Set 57.4 on D, to the diameter in inches (16) on C; and againſt the diameter (16) in inches, is the content (1.24) in feet.

3. *Or thus, by the Rule.*

Set 7.7 on D, to the circumference (50.3) on C, and againſt (50.3) on D, is the content (2140) on C, in the ſame meaſure.

4. *Or thus, by the Rule.*

Set 320 on D, to the circumference in inches (50.3) on C, and againſt the circumference (50.3) on D, is the content in feet (1.24) on C.

Cor. *To find the ſolidity of an Orb.*

This is only finding the ſolidities of the two ſpheres, and ſubtracting the inner from the outer.

SCHOLIUM.

The number .52359 is the ſolid content of a ſphere, whoſe diameter is 1.

And .0169 is the content, when the circumference is 1.

And $1.3819 = \sqrt{\frac{1}{.52359}}$.

And $57.444 = \sqrt{\frac{1728}{.52359}}$.

And $7.6923 = \sqrt{\frac{1}{.0169}}$.

And $319.76 = \sqrt{\frac{1728}{.0169}}$.

Fig. 51.

PROB. XV.

To find the content of the ſegment of a ſphere ADB.

1. RULE.

To thrice the ſquare of the baſe AB, add four times the ſquare of the height DC, divide the ſum by 6; multiply the quotient by the height DC, and the product by .7854 for the ſolidity. This holds for either ſegment ADB or ANB.

2. *Otherwiſe.*

From ſix times the diameter DN, ſubtract four times the height DC; multiply $\frac{1}{3}$ of the remainder by the ſquare of the height DC, and that product by .7854, for the content; for either ſegment, ADB or ANB.

3. *Otherwiſe.*

$\frac{9}{16}$ baſe × height = content, but leſs exact. This ſerves only for the leſſer ſegment.

Examp.

Let AB *be* 28 *inches, and the height* CD, 6½.

By Logarithms.

28	1.447158	2352	
28	1.447158	169	
3	0.477121		
		2521	3.401573
2352	3.371437	6.5	0.812913
		.7854,	—1.895091
6.5	0.812913		
6.5	0.812913		4.109577
4	0.602060	6,	0.778151
169	2.227886	2145,	3.331426
		Content.	

Fig. 52.

PROB. XVI.

To find the ſolidity of any ſpherical Fruſtum or Zone, BACD.

1. RULE.

To the ſum of the ſquares of the diameters of the baſes, AB, CD; add $\frac{4}{3}$ the ſquare of the height AL; multiply the ſum by half the height AL, and the product by .7854 gives the content.

2. *Or thus.*

Multiply half the ſum of the ſquares of baſes AB, CD, by the height AL, to which add $\frac{2}{3}$ the cube of AL: and multiply the whole by .7854 for the content.

Examp.

Let AB *be* 24, CD 42, *and the height* AL 21.

24	42	21	441
24	42	21	4
96	84	21	3)1764
48	168	42	588
576	1764	441	

576
1764
588

Log: 2928,		3.466571
Log: $\frac{1}{2}$AL,	10.5	1.021189
Log: .7854	—	1.895091
Content, 24147	—	4.382851

PROB.

Fig. 53.

PROB. XVII.

To find the ſolidity of a Spheroid ACBD.

RULE.

Multiply the ſquare of the diameter of the ſpheroid AB, by $\frac{2}{3}$ the axis CD; the product multiply'd by .7854 gives the ſolidity.

Examp.

Let the diameter AB *be* 34, *and axis* CD 60.

Log : AB	34	1.531479
Log :	34	1.531479
Log : $\frac{2}{3}$CD,	40	1.602060
Log :	.7854	1.895091
Content	36317,	— 4.560109

PROB. XVIII.

To find the ſolid content of the ſegment of a Spheroid ADB.

1. RULE.

54. Multiply twice the ſquare of the conjugate diameter, by the height of the ſegment, and divide the product by the tranſverſe diameter; to this quotient, add the ſquare of the diameter of the baſe; and multiply the ſum by $\frac{1}{3}$ the height, and then by .7854, for the content.

2. *Or thus.*

Let $2r$ be the tranſverſe, B the baſe of the ſegment, b the height; then $\frac{3r-b}{2r-b} \times \frac{1}{3}Bb =$ ſolid content.

That is, ſubtract the height of the ſegment from Fig. $\frac{2}{3}$ the tranſverſe axis, by this multiply the ſquare of the diameter of the baſe, for a dividend. Subtract the height from the tranſverſe, for a diviſor. Multiply the quotient by $\frac{1}{3}$ the height, and then by .7854.

3. *Otherwiſe.*

When you have not the tranſverſe and conjugate 53. diameters; multiply the area of the baſe by $\frac{9}{16}$ of the height, for the content of the leſſer ſegment.

Examp.

Let the axis be $36\frac{2}{3}$, the conjugate 18, the baſe AB 14, and height CD $6\frac{1}{2}$.

18
18

144
18

324
2

648
$6\frac{1}{2}$

3888
324

4212

36.66)4212.0(114.9
36666

5454
3666

1788
1466

322
324

14
14

56
14

196
114.9

3)310.9
103.6
$6\frac{1}{2}$

6216
518

673.4

673.4
.7854

47138
5387
337
27

528.89
content.

Fig.

PROB. XIX.

To find the content of the fruſtum or middle zone of a Spheroid, adjoining to the center.

RULE.

55. To twice the ſquare of the diameter of the greater end CD, add the ſquare of the diameter of the leſſer end AB; multiply the ſum by $\frac{1}{3}$ the height AF; and the product by .7854 for the content.

Examp.

Suppoſe the baſe CD 18, AB 14, *height* EF 12.

```
 18     14    844
 18     14      4 = 1/3 height,
 ---    ---   ----
144     56    3376
 18     14    ----
 ---    ---
324    196
   2    648
 ---    ---
648    844
        ---

Log : 3376        3.528402
Log : .7854     —1.895091
                ----------
Content 2651½, —3.423493
                ----------
```

PROB.

Fig. 56.

PROB. XX.

To meaſure a Paraboloid, or parabolic Conoid ACB.

1. RULE.

This is generated by a parabola revolving round its axis CD.

Multiply the ſquare of the diameter of the baſe AB, by half the height CD; and the product by .7854 for the content.

2. *Or thus.*

Multiply the area of the baſe by half the height.

Example.

Let the diameter of the baſe AB *be* 32, *and height* CD 22.85.

Log : 32	—	1.505150
Log : 32	—	1.505150
Log : ½ height 11.42,		1.057666
.7894	—	1.895091
Content 9184½,		3.963057

By the Rule.

Set 1.13 on D, to half the height (11.4) on C; and againſt the diameter (32) on D, is the content (9180) on C.

PROB. XXI.

To meaſure the fruſtum of a Paraboloid, AEFB.

RULE.

Add the ſquares of the diameters of the baſes together; multiply the ſum by half the height, and the product by .7854 for the content.

Examp.

Fig. 56.

Examp.

Suppoſe the greater diameter 32, *the leſſer* EF 26, *and height* OE 8.

32	26	1024	.7854
32	26	676	6800
64	156	1700	47124
96	52	4 = ½ height.	6283
1024	676	6800	5340.7 content.

PROB. XXII.

To meaſure a parabolic Spindle ABD.

RULE.

Multiply the ſquare of the diameter AB, by $\frac{8}{15}$ 57. the length CD, and the product by .7854 for the ſolidity.

Note, This is formed by revolving about the ordinate CD of a parabola.

Examp.

Let the baſe AB *be* 24, *the height* CD 37.52.

Log : AB 24	1.380211
Log : AB 24	1.380211
Log : .7854	—1.895091
Log : 8	0.903090
Log : 37.52	1.574263
	5.132866
Log : 15	1.176091
Content 9053,	3.956775

Fig. 57.

PROB. XXIII.

To meaſure the fruſtum of a parabolic Spindle AEFB.

RULE.

To 8 times the ſquare of the greater diameter AB, add 3 times the ſquare of the leſſer diameter EF, and 4 times the product of the two diameters AB, EF. Multiply the ſum by $\frac{1}{15}$ the length EL; and that product by .7854, for the content.

Examp.

Let AB *be* 24, EF 17, *and height* EL 20.

24	17	24	4608
24	17	17	867
			1632
96	119	168	
48	17	24	7107
576	289	408	
8	3	4	
4608	867	1632	

L : 7107	3.85168
L : 20	1.30103
L : .7854	— 1.89509
	5.04780
L : 15	1.17609
Content, 7442½,	3.87171

PROB.

Fig. 58.

PROB. XXIV.

To find the content of a Hyperboloid, or hyperbolical Conoid, ACB.

A hyperboloid is generated by a hyperbola ACB, revolving about its axis CD; and the hyperbola is one of the Conic Sections; to meaſure it.

1. RULE.

Let a = tranſverſe axis, round which it revolves; b = height; B = area of the baſe.

Then $\frac{3a + 2b}{6a + 6b} \times Bb$ = ſolid content. That is,

To 3 times the tranſverſe, add twice the height of the ſolid; multiply this into the ſquare of the diameter of the baſe, which multiply by the height, and then by .7854; reſerve the product for a dividend. To 6 times the tranſverſe add 6 times the height, for a diviſor; the quotient is the content.

2. *Or thus.*

When you know not the tranſverſe; multiply the ſquare of the diameter of the baſe by $\frac{5}{12}$ the height, and then by .7854, for the content, nearly.

Examp.

Examp.

Let the tranſverſe be 20, *the height* CD *of the ſolid* 6, *the diameter* AB *of the baſe* 9.

```
 20    60       9        5832
  3    12       9           6
 ——    ——      ——      ——————
 60    72      81   156)34992(224.3
 ——            72       312...7854
  6           ————      ———   ————
  2           162        379  15701
 ——           567        312   1794
 12           ————       ———    112
 ——————       5832       672      9
 20     6     ————       624
  6     6                ———  176.16
 ——    ——                 48  content
120    36                ———
 36    ——
 ——
156
 ——
```

If the content be found by the other rule, it comes out only 159.

Cor. *If the fruſtum of a hyperboloid was required, it may be meaſured as the fruſtum of a cone. For in moſt caſes it differs ſo little from it, that the one may be taken for the other, there being no ſenſible difference.*

SCHOL.

If both the tranſverſe and conjugate axes of the hyperboloid were given; the ſolidity may be found by the very ſame rule, as for that of the ſegment of a ſpheroid, in Prob. 18; which is Rule 1.

PROB.

Fig. 59.

PROB. XXV.

To find the ſolid content of a circular ſpindle ABD, *or its fruſtum; which is a ſolid generated by the ſegment of a circle, revolving about the baſe* CD.

RULE.

Let D = diameter of the circle AF; *d* = MG, the top diameter of the fruſtum; *x* = height PC; A = arch AM; *b* = BF, the difference between the diameter AF and baſe AB.

Then $x \times \overline{DD - \frac{4}{3}xx} \mp b \times \overline{dx + DA} : \times .7854$, is the ſolid content; according as AB is leſſer or greater than AF. That is,

From the ſquare of the diameter of the circle, ſubtract $\frac{4}{3}$ the ſquare of the fruſtum's height; which multiply by the height, and reſerve the product.

Then multiply the top diameter by the height; alſo multiply the diameter of the circle, by the arch; and add it to the former; and multiply the ſum, by the difference between the diameter and baſe; ſubtract this product from the firſt product, when the baſe of the fruſtum is leſs than the diameter of the circle; or add it, when greater; and multiply the reſult by .7854.

When the whole ſpindle is required, the part *dx* is nothing.

And the part MDG may be found, by ſubtracting AMGB from the whole.

Examp.

Fig. 59.

Examp.

Let the diameter AF *be* 20, AB 13, MG 10; CP 5.26;

From the radius and height CP being given, the length of MA will be found 5.54 = A.

$D = 20$	$x = 5.26$	27.667	400
20	5.26	$\frac{1}{3}$, 9.222	36.89
400	3156	36.889	363.11
	1052		5.26
	2630		
			181555
	27.6676		7262
			2178
		1 Prod.	1909.95

$x = 5.26$,	$A = 5.54$	1909.9
$d = 10$	$D = 20$	1143.8
52.6	110.8	766.1
	52.6	.7854
	163.4	53627
	$BF = b = 7$	6129
		383
	1143.8	30
	Content,	601.69

SCHOL.

If you deſire to know the ſurface of this ſolid.

It is $\overline{Dx \mp b}\, A \times 3.1416 =$ ſurface of the fruſtum AMGB, according as AB is leſſer or greater than AF.

PROB.

Fig. 60.

PROB. XXVI.

To find the ſolidity of any body CH, *generated by revolving about an axis.*

1. RULE.

Take the diameters of the body, at both ends, and in the middle of the height, as at A, B and I; and let them be CD, GH and EF. Then $\frac{CD^2 + GH^2 + 4EF^2}{6}$ × length AB × .7854, is the ſolid content.

2. *Or thus.*

Find the area of the Section, at the top, bottom, and middle, which let be A, B, and I. Then $\frac{A + B + 4I}{6}$ × AB the length, is the content.

If the body be long, divide it into two or more parts, and meaſure each part by the foregoing rules; and the ſum of them gives the ſolid content.

3. *Otherwiſe.*

61. If the body be ſuch as you can ſee it on all ſides, and at any diſtance; cut a paper, as *agbh*, into the apparent form of the body; and holding it directly between your eye and the body AGH; clip away any ſuperfluous parts, from the ſides of the paper, till it juſt cover the body, and their edges coincide. Then if the baſe or height GH or AB, be meaſured, and the baſe or height *gb* or *ab* of your paper be made the ſame, by a proper ſcale; then you may find as many diameters, on your paper, as you will, by the ſame ſcale. From whence, by the former rules, you may find the ſolidity of the body generated by your paper ſcheme, which will be the ſame as that of the great body;

as they are ſimilar ſolids. And thus ſuch a body as a hay ſtack may be meaſured. Fig. 60. 61.

Examp.

Suppoſe the diameters CD, EF, *and* GH, *be found to be* 12, 24 *and* 30, *and height* AB 32.

```
 12      24      30       576      .7854
 12      24      30         4        630
----    ----    ----     -----    ---------
144      96     900      2304     235620
----     48     ----      900     47124
        ----              576     ---------
        576              -----    494.8020
        ----          6)3780      ---------
                          630
494.8
  32
-----
 9896
14844
-------
15833.6  content.
-------
```

PROB. XXVIII.

To find the content of a very irregular body.

1. RULE.

Procure a priſmatic or cylindric veſſel that will hold the body. Put in the body, and pour in water to cover it; make a mark at top of the water, and then take out the body; and obſerve how far the water has deſcended; meaſure that ſpace of the veſſel correſpondent to that deſcent; and you have the ſolidity of the body. If the veſſel will not hold water, you may uſe ſand inſtead of water.

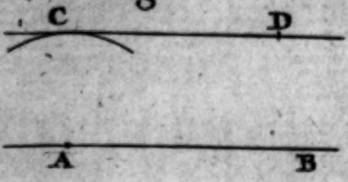
And thus a fragment or parcel of a body may be meaſured without dividing it from the whole; and that by immerſing only the fragment, and taking it out again, meaſure the deſcent of the water, as before.

Or if your veſſel be irregular, fill it with water, then put in your body; and weigh the water that runs over. Whence the content of the water, or of the body, will be found, by the tables of ſpecific gravity before given.

2. *Otherwiſe.*

Find what a cubic inch or a cubic foot of the ſame matter weighs, either by trials, or from the tables of ſpecific gravity. Then weigh the irregular body, and its content will be known, becauſe the ſolidities are as the weights. For if you divide the weight of the body by the weight of an inch or foot, the quotient will be the ſolidity in inches or feet.

Fig:1.



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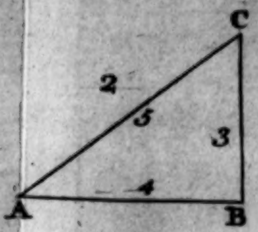

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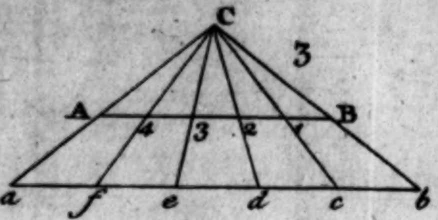

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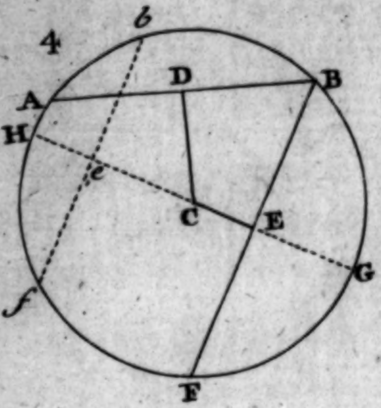

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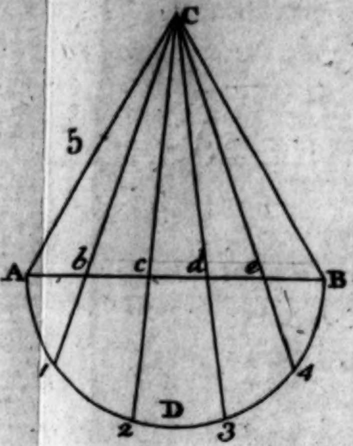

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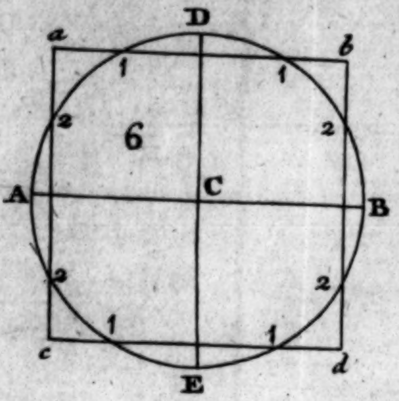

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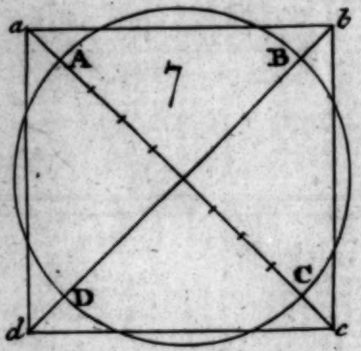

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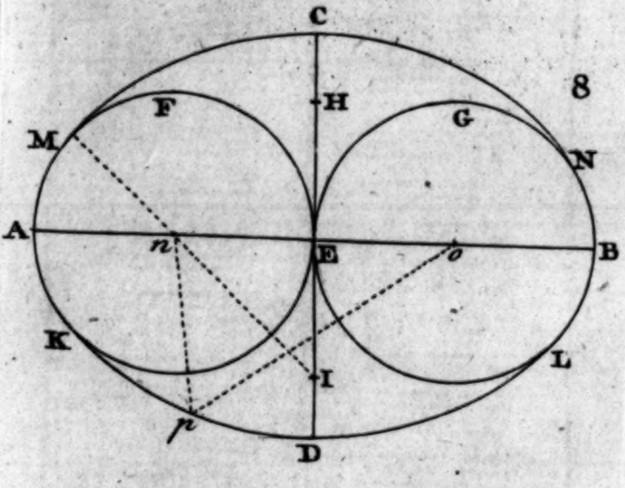

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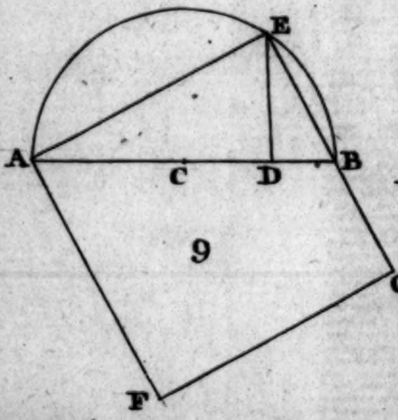

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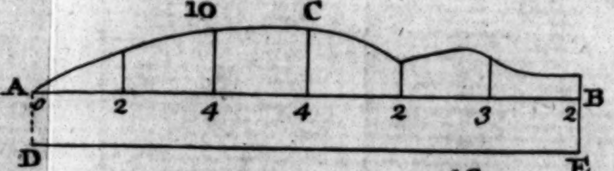

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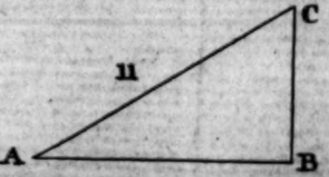


Mensuration

PART V.

GAUGING.

GAUGING is a branch of meaſuring ſolids, and teaches how to find the contents of all ſorts of veſſels, in gallons, or ſome meaſure of capacity. And this differs from other kinds of meaſuring, where the meaſures may be taken, and contents given in feet, yards, &c. But here the meaſures are always taken in inches, or at leaſt muſt be reduced to inches, and the contents given in gallons; for which purpoſe, certain fixt numbers are made uſe of to facilitate the computation, as will be ſhewn afterwards. But in the gauging of malt, the content muſt be had in buſhels.

And in finding the areas of all plane ſurfaces in gauging, their contents are alſo expreſſed in gallons; becauſe in the buſineſs of gauging, every area or ſurface is ſuppoſed to be one inch deep; and therefore in the Gauger's language, the area of ſuch a ſquare, or ſuch a circle, is ſo many gallons. For there will be the ſame number of ſuperficial inches in any plane figure, as there will be ſolid inches of one inch depth; ſo that what is called a ſurface in gauging, is in reality a ſolid, whoſe height is one inch.

As the ſliding rule is of great ſervice in the art of gauging, and very much uſed; I ſhall here deſcribe the principal parts of it, as it differs from the other ſort, deſcribed in the ſecond Part.

Of the SLIDING RULE.

The Gauger's Sliding Rule differs in many things from that before deſcribed, called Cogſhal's Sliding Rule. Upon this rule there are alſo three double lines of numbers, called A, B, C; and a ſingle line called D; this line is marked from the beginning 1, 2, 3, &c. to 10 at the end. And there is alſo a triple line of numbers called E, for extracting the cube root, &c. All theſe lines begin and end together. They work by the means of three ſliders, moving in three faces of the rule. And theſe lines are logarithmic lines, ſince they all anſwer to multiplication and diviſion. Each part of the line E is a third part of the correſpondent diſtance on D.

There is a line marked MD, (*malt depth*), the ſame with A or B, but inverted; it begins at 21.5, and ends at 2.15. This line is alſo logarithmical. Its uſe is for gauging malt. Here 2150.4 on MD is oppoſite to 1 on B; and 1 on MD is oppoſite to 2150.4 on B or A, at a point marked MB.

On the other ſide, by the line of inches, are 3 varieties of caſks; the uſe of theſe, is to find a mean diameter, to reduce the caſk to a cylinder. The practice is thus; take the difference of the head and bung diameters, and find it in the ſcale of inches, and againſt it (in the proper variety) you will have a number, to be added to the head diameter, for the mean diameter.

On the inſide of the ſliders are placed two lines, one numbered from 13 to 16, the other from .42 to 3.61 gallons; and ſerves to ſhew the area in ale gallons, for any diameter. The uſe is this; one ſlider is to be pinned faſt at the end towards the right hand; and the other ſlider is drawn out, till the two ends touch the oppoſite ſides of the tub;

then the diviſions, cut by the end of the rule, Fig. ſhew the diameter on one line; and the area in gallons on the other. When the diameter is more than 24½, draw out the other ſlider, which will ſhew the like.

On another ſide of the rule is a ſlider, with a line of numbers C, like A and B, againſt which are two lines SL (*ſegments lying*), and SS (*ſegments ſtanding*); theſe are for finding the ullage of a caſk in ale. The uſe of theſe lines is this. Find the bung diameter when the caſt lies, or the length when it ſtands. Then

Set the bung diameter on C, to 100 on SL for a lying caſk; or ſet the length on C, to 100 on SS for a ſtanding caſk; and againſt the wet inches on C, is a ſegment on SL, or SS.

Then ſet 100 on A, to the caſks content on B; and againſt that ſegment on A, is the ullage (or liquor in the caſk) on B.

On the line D, are ſeveral marks, which ſhew the gauge points; as WG (*wine gallons*), AG (*ale gallons*), MS (*malt ſquare*), MR (*malt round*). What theſe are, and where to be placed, will be ſhewn in the firſt Prob.

On the plane ſide of the rule, there are ſeveral lines, as one marked FM (*foot meaſure*), where a foot is divided into 100 parts. And another marked IM (*inch meaſure*) being a ſcale of inches. And another marked FC (*fruſtum of a cone*), which ſerves to meaſure its content.

To extract the cube root by the rule. Set the lines D, E, even; that is, 1 againſt 1; then againſt the given number on E, is the cube root on D.

There is a line on the gauging rod, called the *diagonal line*, for finding the content of a caſk; which is thus. Put the rod into the bung hole, to either oppoſite angle; and mark the number at the middle of the bung hole, for that is the content of the caſk.

PROB. I.

To find proper multipliers, diviſors, and gauge points.

1. *For ſquare figures.*

Since 282 inches make an ale gallon, 231 inches a wine gallon, and 2150.4 inches a malt buſhel, therefore when the content of any figure or veſſel is had in inches; 282, 231, and 2150.4 will be the diviſors, for reducing it to *ale gallons*, *wine gallons*, or *malt buſhels* reſpectively.

But if inſtead of theſe diviſors, you would chuſe to have multipliers; then theſe will be had by dividing 1 with a number of cyphers, by 282, 231, and 2150.4 reſpectively. Thus

282)1.000000(.0035461 *for ale gallons.*
231)1.00000(.0043289 *for wine gallons.*
2150.4)1.00000(.0004651 *for malt buſhels.*

The gauge points for ſquare figures are no more than the ſquare roots of 282, 231, and 2150.4; which gauge points are therefore the ſide of a ſquare, whoſe area is a gallon or buſhel.

$\sqrt{282} = 16.79$ *ale gallon.*
$\sqrt{231} = 15.19$ *wine gallon.*
$\sqrt{2150.4} = 46.37$, *Wincheſter buſhel.*

For the ſecond gauge points for ſquares, multiply (or divide) the former gauge points by the ſquare root of 10, or 3.162, or thus,

$\sqrt{2820} = 53{,}10$ *ſecond* GP *for ale.*
$\sqrt{2310} = 48.06$ *ſecond* GP *for wine.*
$\sqrt{215.04} = 14.66$ *ſecond* GP *for malt buſhels.*

All gauge points are the firſt terms of any proportion upon the lines D and C; and theſe ſecond gauge points ſerve, when the terms run off the lines.

For the numbers on D, being as the ſquare roots of thoſe on C; the variation of 10 on C, will cauſe a varia variation of the gauge point on D, equal to the ſquare root of 10. And there muſt be ſuch a variation on C, to bring any number upon another radius. Fig.

2. *For circular figures.*

Since .7854, multiplied by the ſquare of the diameter, is the area of a circle in inches; this divided by 282, 231, or 2150.4, will give the content in ale gallons, wine gallons, or malt buſhels. That is $\frac{.7854}{282}$, or $\frac{.7854}{231}$, or $\frac{.7854}{2150.4}$, muſt be multiplied into the ſquares of the diameters. Whence the diviſors will be found by dividing by .7854;

.7854) 282 (359.05 *for ale gallons.*
.7854) 231 (294.12 *for wine gallons.*
.7854) 2150.4 (2738 *for malt buſhels.*

Then the ſquare of the diameter, being divided by any of theſe, gives the circle's area.

But to find the multipliers; theſe are had by dividing .7854 by 282, 231, and 2150.4 reſpectively.

282) .7854 (.002785 *for ale gallons.*
231) .7854 (.0034 *for wine gallons.*
2150.4) .7854 (.000365 *for buſhels.*

The gauge points for circles, are the diameters of ſuch circles as contain ſome known meaſure, as a gallon; as if d be the diameter, then $.7854dd =$ 282 for ale; whence $d = \sqrt{\frac{282}{.7854}}$ for ale, and ſo of the reſt. Or thus,

$\sqrt{359.05} = 18.95$ *the* GP *for ale.*
$\sqrt{294.12} = 17.14$ *the* GP *for wine.*
$\sqrt{2738} = 52.32$ *the* GP *for buſhels.*

The ſecond gauge points are found by multiplying (or dividing) the former by $\sqrt{10}$ or 3.162; or thus,

$\sqrt{3590.5} = 59.92$ *for ale gallons.*
$\sqrt{2941.2} = 54.22$ *for wine gallons.*
$\sqrt{273.8} = 16.54$ *for buſhels.*

SCHOL.

The ſecond gauge points may alſo be found by the rule, thus. Set 1 upon C, to the firſt gauge point upon D; then againſt 10 on C, is the ſecond gauge point required; or elſe againſt $\frac{1}{10}$ upon C, is $\frac{1}{10}$ the gauge point, which multiply by 10.

Examp.

Let the firſt gauge point be 18.95. Set 1 upon C, to 18.95 on D; and againſt $\frac{1}{10}$ on C, is 5.99 on D, which multiply by 10, gives 59.9 the ſecond gauge point.

PROB. II.

To find the area of a ſquare.

RULE.

16. Multiply the ſide AB or AD in inches, by itſelf; and divide the product by the proper diviſor, (in Prob. I.); or multiply it by the proper multiplier.

Examp.

Let the ſide of the ſquare be 42 *inches; to find the area in ale.*

42	*or thus,*
42	.003546
	1764
84	
168	3546
	2482
282) 1764(6.25 ale gallons.	212
	14
	ale gall. 6.254

By the Sliding Rule. 16.

Set 282 upon A to 42 upon B; then againſt 42 on A is 6.25 on B.

PROB. III.

To find the area of any Parallelogram.

RULE.

Multiply the length by the breadth; and divide the product by the proper diviſor (Prob. I.) or multiply it by the proper multiplier. 18.

Examp.

There is an oblong whoſe length AB *is* 48, *and breadth* AE 32; *to find the area in wine.*

48		*or thus,*
32		.004329
——		1536
96		——
144		4329
——		2164
231) 1536(6.65 wine		130
gallons.		26
		——
	wine gall.	6.649
		——

By the Rule.

Set 231 on A to the length 48 on B; and againſt the breadth 32 on A, is 6.65 the area on B.

PROB.

PROB. IV.

To find the area of a Triangle, ACB.

RULE.

14. Multiply the baſe by half the perpendicular, and divide by the proper diviſor (Prob. I.), or multiply by the proper multiplier.

Examp.

Let the baſe AB *be* 154, *the perpendicular* CD 46.2; *to find the area in buſhels.*

154	*or thus.*
23.1	3557.4
	.0004651
154	
462	142296
308	21344
	1779
2150.4)3557.4(1.65	35
21504 buſhels.	
	buſhels 1.65454
14070	

By the Rule.

Set 2150 on A to the baſe 154 on B; and againſt 23.1 on A is the content 1.65 on B.

PROB. V.

To find the area of a Trapezium ABCD.

RULE.

19. Draw the diagonal AC, and the two perpendiculars upon it, BE, DF. Then multiply the diagonal by half the ſum of the perpendiculars, and divide by the proper diviſor (Prob. I.), or multiply by the correſpondent multiplier. *Ex.*

Fig. 19.

Examp.

Suppoſe the diagonal AC 28, *and the perp.* BE 13, *and* DF 24. *To find the area in ale gallons.*

```
   13          28                  or thus,
   24          18.5               .003546
   ——          ————                   518
2) 37           140               ————————
   18.5         224                 17730
  ————          28                    354
                ————                  283
        282) 518.0(1.84 ale       ————————
                   gallons.        1.8367
             ——————————————       ————————
```

By the Rule.

Set 282 on A to 28 on B; and againſt 18.5 on A, is the content 1.84 on B.

Cor. *Hence the area of any irregular poligon will be found, by dividing it into trapeziums or triangles, and meaſuring each of them ſeparately, and taking the ſum of them all.*

And a regular poligon may be meaſured as a triangle, taking the circumference of it for the baſe; and the diſtance of the center from any ſide, for the perpendicular.

PROB. VI.

To find the area of a Circle.

RULE.

Square the diameter, and divide it by its proper diviſor (Prob. I.), or multiply by the correſpondent multiplier. 23.

Fig. 23.

Examp.

If the diameter AC *of a circle be* $54\frac{1}{2}$ *inches; what is the area in wine gallons?*

54.5	*or thus.*
54.5	2970.25
	.0034
2725	
2180	1188100
2725	891075
294.12)2970.25(10.10 w. gallons.	10.098850 w. gallons.

1. *By the Rule.*

Set 17.14 on D, to 1 on C; then againſt $54\frac{1}{2}$ the diameter on D, is the area 10.1 on C.

As the rule is thus ſet, it will be a table of circles areas, in wine gallons; for againſt any diameter on D, will be found its area on C. And the like for ale gallons, uſing the proper gauge point.

2. *Or thus, by the Rule.*

Set 294 on A, to the diameter $54\frac{1}{2}$ on B; and againſt $54\frac{1}{2}$ on A, is the area 10.1 on B.

Cor. *The ſector of a circle is meaſured as a triangle, taking the arch for the baſe, and the radius for the perpendicular.*

PROB. VII.

To find the area of an Ellipſis, or oval figure.

RULE.

Multiply the longeſt and ſhorteſt diameters together, and divide the product by the proper diviſor for circles (Prob. I.), or multiply by the proper multiplier.

Examp.

Fig. 23.

Examp.

If the greateſt diameter be 36.3, *the leaſt* 26.8; *what is the area in ale gallons?*

36.3
26.8

2904
2178
726

359.05)972.84(2.709

or thus,

972.|84
.002785

1945
680
78
4
ale gall. 2.707

By the Rule.

Set 359 on A, to 36.3 on B; and againſt 26.8 on A, is 2.71 the content on B.

PROB. VIII.

To find the area of the ſegment of a Circle ADB.

RULE.

Let D be the vertex, and DE perp. to AB. Multiply the difference between the arch AD and ſine AE, by the radius; alſo multiply the ſine AE by the height DE; and divide the ſum of the products by the proper diviſor for ſquares (Prob. I.), or multiply by the proper multiplier. This ſerves either for the leſſer or greater ſegment. 24.

Note, if the diameter is not known, it may be found by Prob. XVIII. Part II.

Examp

Fig. 24.

Examp.

Suppoſe the radius CA 31.2, *height* DE 20, *arch* AD 37.55, *half the baſe* AE 29.12. *To find the area in ale gallons.*

37.55	29.12	*or thus.*
29.12	20	
		845.4
8.43	582.40	.003546
31.2	263.01	
		25362
1686	282)845.41(3.00 ale gall.	4227
843		338
2529		50
263.016		ale gall. 2.9977

PROB. IX.

To find the area of the ſegment of an Ellipſis.

RULE.

62. Multiply the baſe AB by $\frac{2}{3}$ the height DE, and reſerve the product. Then ſuppoſing C the center of the ellipſis, multiply the former product by the height DE, and then divide by ſeven times the ſemidiameter DC, and add the quotient to the firſt product. Then divide the ſum by the proper diviſor for ſquares, &c.

Examp.

Fig. 62.

Examp.

Suppoſe AB 30, DE 21, DC 40; *to find the area of* ADB *in wine gallons.*

21
2
3)42(14
30
420
40
7
280

420
21
420
840
280)8820(31.5

420
31.5
231)451.5(1.95 wine gall.

PROB. X.

To find the area of a Parabola or Hyperbola.

RULE.

Multiply the baſe AB, by $\frac{2}{3}$ the height DE in the parabola, or by $\frac{5}{8}$ the height in the hyperbola; and divide by the proper diviſor, for ſquares, &c.

Ex.

In a hyperbola let AB *be* 400, DE 280, *to find the area in malt buſhels.*

280
5
8)1400(175.

175
400
2150.4)70000(32.55 buſhels.

Schol.

If the area of any irregular curve is required, it muſt be found by the method laid down in Prob. XIII.

Fig. 62. XIII. Part II. and then divided by the proper diviſor, or multiplied by the correſpondent multiplier, for ſquare figures.

PROB. XI.

To find the content of any Priſm CB.

RULE.

41. 42. Find the area of the baſe AB, by ſome of the foregoing Problems, multiply that by the perpendicular height, and you have the content.

Examp.

There is a priſm whoſe baſe contains 5.21 *ale gallons; and the height is* 20 *inches.*

5.21
20

104.20 ale gallons.

By the Rule.

Set 1 on A to the baſe 5.21 on B; and againſt the height 20 on A, is 104 the content on B.

Or when the baſe is a ſquare.

Set the proper gauge point for ſquares on D, to the height on C; then againſt the ſide of the ſquare on D, is the content on C.

PROB. XII.

To find the content of a Cylinder.

RULE.

39. Find the area of the baſe by Prob. VI. and multiply it by the height.

Examp.

Fig. 39.

Example. 1.

The diameter of a cylinder is $54\frac{1}{2}$, *and height* 14 *inches; to find the content in wine gallons.*

The baſe contains 10.1 wine gallons, therefore,

$$\begin{array}{r} 10.1 \\ 14 \\ \hline 404 \\ 101 \\ \hline 141.4 \end{array} \text{ wine gallons.}$$

By the Rule.

Set 17.14 on D, to 14 the height on C; and againſt the diameter $54\frac{1}{2}$ on D, is the content on C 141 gallons.

Ex. 2.

Let the diameter of a cylinder be 40 *inches, and the depth* 30; *to find the content in ale gallons.*

By Prob. VI, the area of the baſe is 4.457 gal. Therefore,

$$\begin{array}{r} 4.457 \\ 30 \\ \hline 133.710 \end{array} \text{ ale gall.}$$

By the Rule.

Set 18.94 on D, to the height 30 on C; and againſt the diameter 40 on D, you have the content 134 on C.

PROB. XIII.

To find the content of a Pyramid, ABD.

RULE.

44. Find the area of the baſe by the foregoing Problems, and multiply it by $\frac{1}{3}$ the height, gives the content.

Exam.

Suppoſe the area of the baſe 2025 *inches, or* .941 *buſhels, and height* 20; *to find the content in buſhels.*

3)20(6.66 .941
20

18.820 Buſhels.

By the Rule.

Set 1 upon A, to the height 20 upon B; and againſt the baſe .94 on A, is the content 18.8 buſh. on B.

Or when the baſe is a ſquare.

Set 46.37 on D, to the height 20 on C; and againſt the ſide of the baſe 45 on D, is the content 18.8 on C.

PROB. XIV.

33. *To find the content of a cone, ſphere, ſpheroid, pa-*
43. *raboloid, hyperboloid, or parabolic Spindle.*
53.
56. RULE.
57. Find the area of the baſe or greateſt circle, by
58. Prob. VI. in gallons, &c. Then multiply this by ſuch a part of the altitude as here ſet down; for the cone $\frac{1}{3}$, for the ſphere and ſpheroid $\frac{2}{3}$, for the paraboloid $\frac{1}{2}$, hyperboloid $\frac{5}{12}$, parabolic ſpindle $\frac{8}{15}$ of the height.

Fig. 33, &c.

Ex.

Suppoſe a parabolic conoid, the diameter of the baſe 30, the height 26; to find the content in ale gallons.

The area of the baſe is 2.506 gallons, then

2.506
$\frac{1}{2}$ height 13

7518
2506

32.578 ale gall.

By the Rule.

Set 18.95 on D, to half the height 13 on C; and againſt the diameter 30 on D, is the content 32.5 on C.

PROB. XV.

To find the content of the fruſtum of a pyramid, cone, ſphere, ſpheroid, priſmoid, paraboloid, parabolic Spindle, or circular Spindle; or any ſegment thereof. 45. 46. 51, &c.

RULE.

Compute the contents thereof in inches, by the Problems in Part IV; and divide by 282, 231, or 2150.4, for ale gallons, wine gallons, or malt buſhels, reſpectively.

But in circular bodies, it is better to work by the diameters, and the fixt diviſors or multipliers for circles.

Fig. 45, &c.

Examp.

There is a fruſtum of a cone whoſe height is 28 inches, and the diameter of the leſſer baſe is 22 inches, of the greater 32; what is the content in ale gallons?

The area of the leſſer baſe is 379.7, and of the greater 804.25; then the mean area is 552.6; then

$$\begin{array}{r} 379.7 \\ 552.6 \\ 804.25 \\ \hline 1736.55 \\ \hline \end{array} \qquad \begin{array}{r} 1736.55 \\ 9\frac{1}{3} \\ \hline 1562895 \\ 57885 \\ \hline \end{array}$$

282)16207.80(57.47 ale gallons.

Or thus.

$$\frac{22^2 + 22 \times 32 + 32^2}{359} \times 9\tfrac{1}{3} = 57.46 \text{ the content.}$$

1. *By the Rule.*

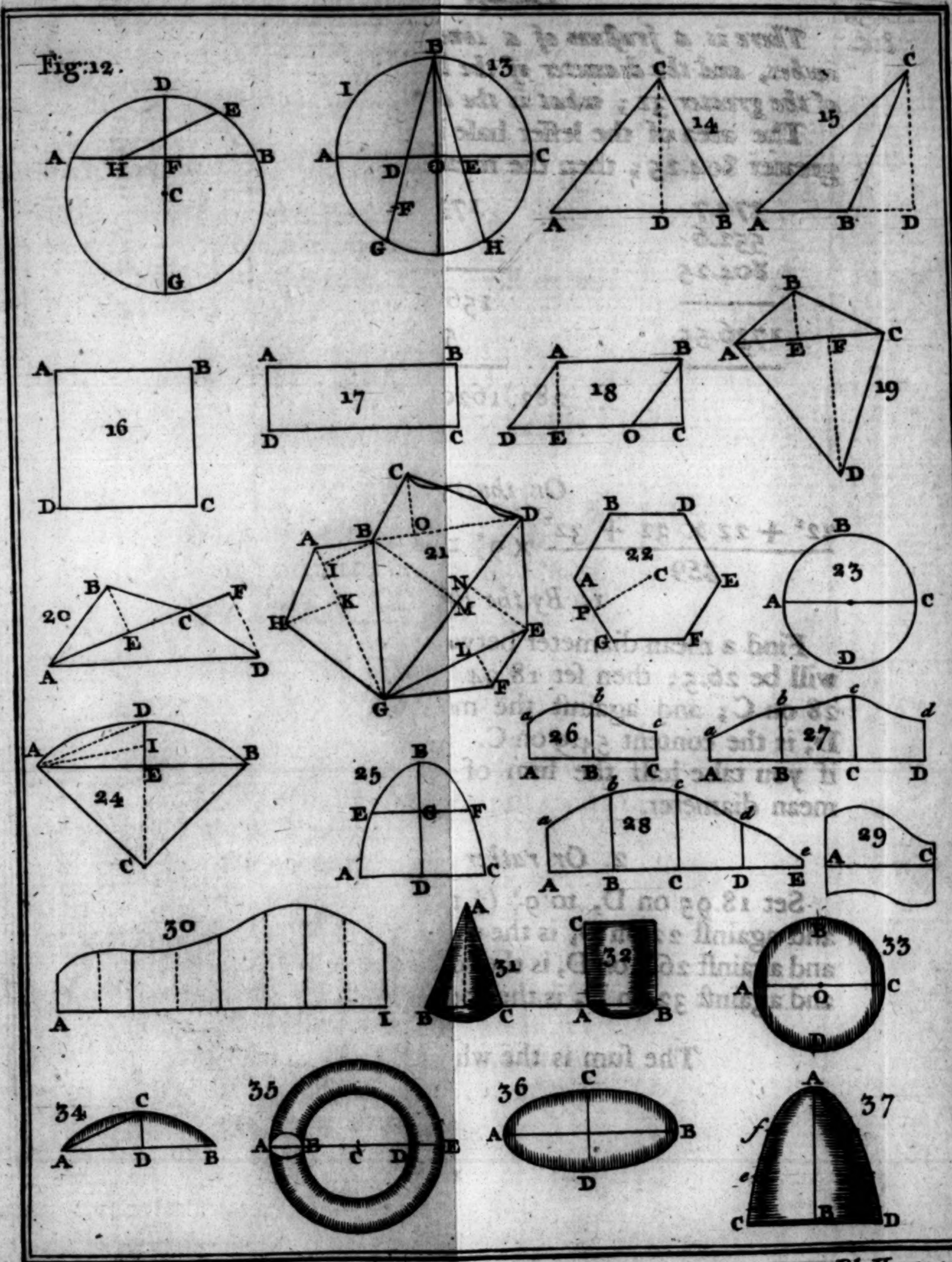
Find a mean diameter between 22 and 32, which will be 26.5; then ſet 18.94 on D, to the height 28 on C; and againſt the mean diameter 26.5 on D, is the content 54.9 on C. But it will be nearer, if you take half the ſum of the diameters for the mean diameter.

2. *Or rather thus.*

Set 18.95 on D, to $9\frac{1}{3}$ ($\frac{1}{3}$ the height) on C.
and againſt 22 on D, is the content on C, 12.6
and againſt 26.5 on D, is the content on C, 18.3
and againſt 32 on D, is the content on C, 26.4

The ſum is the whole content 57.3 gall.

Cor.



Mensuration

Pl: II. pa: 148.

Cor. *If a ſquare fruſtum was propoſed, then the* Fig. *areas of the baſes would be two ſquares, and the con-* 45, *tent is eaſily had thus,* $\frac{22^2 + 22 \times 32 + 32^2}{282} \times 9\frac{1}{3}$ &c.
$=$ content.

That is, to the ſquares of the baſes, add their product, and multiply the ſum by $\frac{1}{3}$ the height, which divide by 282 for the content.

And in working with the rule, 16.79 will be the gauge point for ale.

PROB. XVI.

To gauge any tun or ſolid body whatever, AD.

1. RULE.

Let AB be the diameter at top, CD the diame- 63.
ter at bottom, GHI the diameter in the middle of the depth EF. Then $\frac{AB^2 + CD^2 + 4GI^2}{6} \times$ depth EF $=$ ſolid content; and that being divided by the proper diviſor (Prob. I.), will give the content in gallons.

If the veſſel or tun be high, divide it into two or more parts, and find a middle diameter for each part, and compute them ſeparately by this rule; and the ſum of all is the content.

2. *Or thus.*

Begin at the bottom of the tun CD, and divide 64.
the altitude FE in parts of 10 inches each; and in the middle of every ten inches, take the meaſures of the diameters OO, LL, II, HH; and from thence find all the areas in gallons, at theſe places; then the ſum of all theſe areas, multiplied by 10, will give the content.

Fig. 64. If any odd inches to ſpare at the top, then find likewiſe the area at top, and multiply it by theſe odd inches, to be added to the former.

If the ſections of the tun are not circular; then you muſt take croſs diameters; and take half the ſum, for the mean; or when the difference is great, multiply them together, and the product is the ſquare of a mean diameter, to be made uſe of.

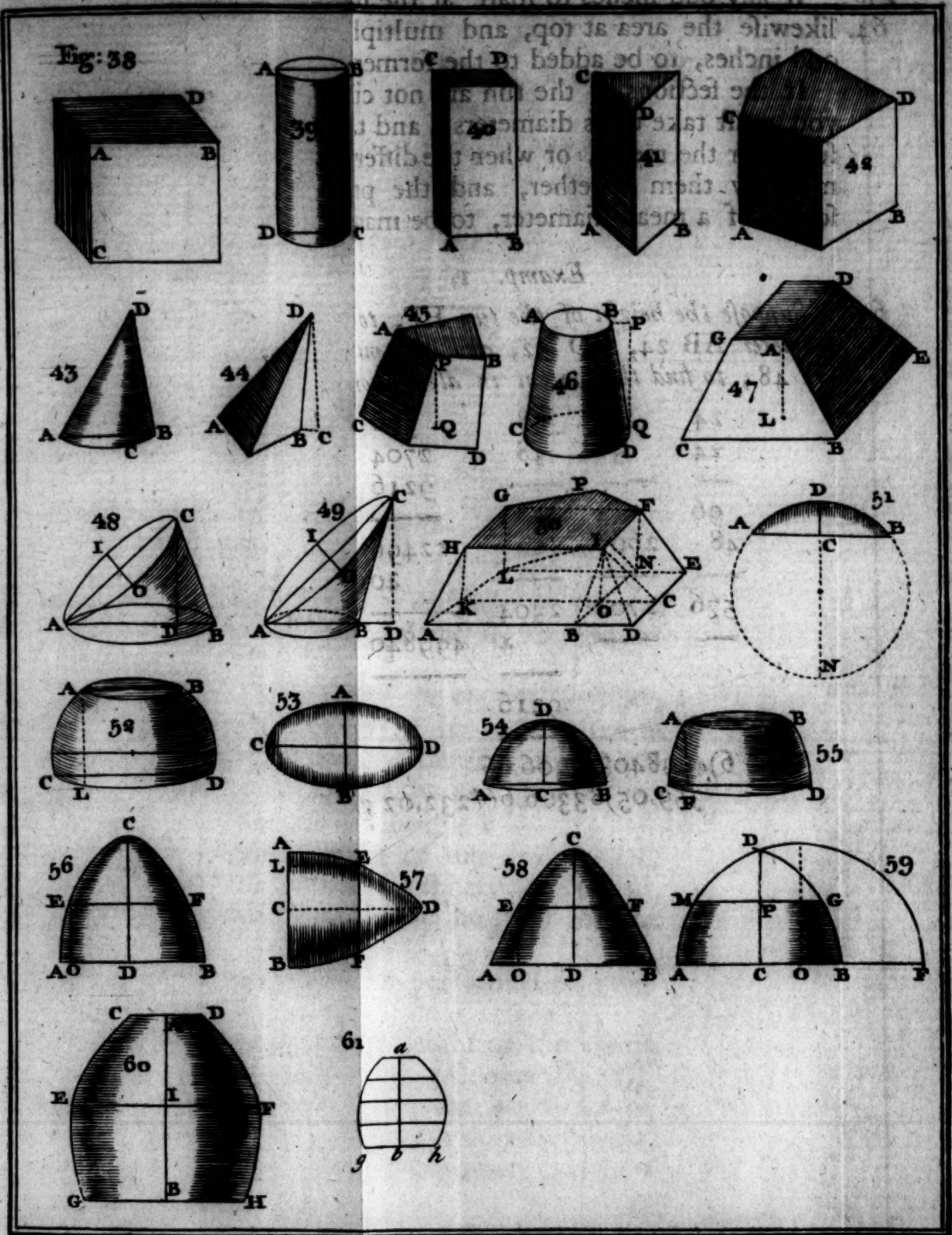
Examp. 1.

63. *Suppoſe the height of the tun* EF, *to be* 40 *inches; diameter* AB 24, CD 52, *and the middle diameter* GI 48; *to find the content in ale gallons.*

24	52	48	576
24	52	48	2704
96	104	384	9216
48	260	192	12496
576	2704	2304	40
		4	499840
		9216	

6)499840(83306.66

359.05)83306.66(232.02 gall.



Mensuration

Pl: III. pa: 150.

Fig. 64.

Examp. 2.

Let EF *be* 40, CD 24, AB 52; *and the diameters taken at* 10 *inches diſtance,* OO 30, LL 46, II 50, HH 51; *to find the ale gallons in it.*

```
  30      46     50      51     900
  30      46     50      51    2116
 ----    ----   ----    ----   2500
 900     276   2500      51    2601
 ----   184     ----   255     ----
        ----           ----    8117
        2116           2601    ----
        ----           ----

359)8117(22.61
           10
        -----
        226.1 gallons.
        -----
```

Cor. 1. *If it be a copper you are gauging, you muſt gauge the crown* CTD, *and ſubtract it from the former content.*

To do this, obſerve, when the copper is empty, the ſhape of the crown. If it be more curve at top than towards the bottom, meaſure it as a paraboloid; and its content will be had by multiplying the baſe CD by half the height FT.

If it have equal curvature at top and bottom, meaſure it as the ſegment of a ſphere, or take $\frac{9}{16}$ of the baſe, multiplied by the height, for the content.

The like operations may be performed in gauging a ſtill.

To find the diameter and height of the crown CD, FT; lay a ſtreight ruler AB over the top, and put down another NC into the hollow at C, and move it till it is perpendicular as at N. Then meaſure AN, and NC; in like manner put down

Fig. the rule at D, and find its diſtance from B, when 64. plumb. Theſe two diſtances from the ſides A and B, added together, and ſubtracted from AB, gives CD. Likewiſe put down a rule from E to T, and meaſure ET, which being ſubtracted from NC, gives TF.

When the copper is empty, if you meaſure as much liquor into it, as will juſt cover the crown, and note it down; then you need not afterward trouble yourſelf about the crown, but meaſure any part above OTO.

65. Cor. 2. *When any tun or veſſel ſtands not lever, the quantity of liquor* CDE, *which will juſt cover the bottom, muſt be computed ſeparately, and is called the drip or fall of the tun.*

Since the depth of the part, at D is but ſmall, it may be reckoned the hoof of a conic fruſtum; and ſo may be meaſured by Prob. VIII. Part IV. But an eaſier way will be to meaſure into the veſſel when empty, ſo much liquor as will riſe to the angle C, or the line CE; and note down the quantity. Alſo at the ſame time you will find the level CE, which you may mark. Alſo you will find the depth at D, which likewiſe ſet down, which will be of future ſervice. For at any time, by ſetting a rod upright at D, you may know how deep the liquor is above the level CE.

The quantity of the liquor CDE will be nearly equal to half the baſe CD × height DE, if DE be perp. to CD.

Cor. 3. *If* ABCD *be a cooler, you muſt find the true place of the dip, in order to get the depth of the liquor at any time.*

To do this, take ten dips in different places of the cooler, when there is but little liquor in it; divide the ſum of them by ten for the mean depth.

If this is not exact enough, take ten more, for another mean depth; then half the ſum may be taken for the exact mean depth. Having the mean depth, find out a place on the ſide, and mark it, where the depth is juſt ſo much, and that will be the dipping place required. Fig. 65.

SCHOLIUM.

For taking the dimenſions of any veſſel; by help of the ſliding rule, take croſs diameters at the wideſt and ſtraiteſt places; and if they differ, take a mean between them; and that is the diameter in that place. This muſt be done at top, and bottom, and as many places as are neceſſary, if the tun be irregular. Set the rule to the mean diameter, and find where that diameter agrees with the tun at that depth. From theſe points, at the ends of the rule, draw chalk lines up and down. On theſe you may ſet any altitude, and draw croſs lines, through the points of diviſion.

In inching any tun, a chalk line is to be drawn through the place of the mean diameter, up and down; upon this every inch muſt be ſet, from the top or bottom. If the ſide of the tun is not perpendicular, allowance muſt be made in the inching.

When the top of a vat is covered with yeaſt; tye a cork (loaded with lead) to a ſtring, and let it down thro' the yeaſt, to ſwim upon the liquor. Then put down a rod to the bottom, cloſe to it; and taking them out together, you will ſee where the top of the cork touches the rod, for that place ſhews the depth.

PROB.

Fig. 66.

PROB. XVII.

To inch any Ton or Veſſel.

The *inching* of a tun or veſſel, is finding how much liquor it will hold at every inch of its depth. In order to this, take this,

I. RULE.

Divide the tun into ſeveral parts or fruſtums, each of ten inches deep; and find the content of every fruſtum, by taking diameters in the middle of them, as was ſhewn in the laſt Prob. And make a ſmall table, ſhewing the mean diameter, the area, and the content of each fruſtum; and the total ſum, or content of the tun.

Then by help of this, make another table, ſetting down the depth at top, againſt the content; and ſet down the depth of the liquor inch by inch, decreaſing from the whole depth to 0; againſt each of which the correſponding area is to be placed, which is found thus.

Take the area for the mean diameter of the top fruſtum, and write it on a ſcrip of paper, and ſubtract it from the whole content, and ſet down the remainder, and then ſubtract it from that remainder, and ſet down the next remainder, and ſo on; continuing to ſubtract as oft as there are inches in the top fruſtum; ſetting them down againſt their reſpective depths.

Then take the next mean area, and ſubtract continually as before, till the next fruſtum be done. And the like for all the fruſtums. And at laſt there will remain 0, if the work be right.

Examp.

Fig. 66.

Examp.

Suppoſe ABDC *is a tun to be inched, whoſe height is* 23 *inches, the diameter* CD 24, OO 30, LL 46, AB 48; *to find what ale gallons it holds at every inch.*

Here the diameter OO is in the middle of the loweſt 10 inches, LL in the middle of the next 10 inches, &c. Then proceeding by the former directions, and the laſt Prob. we have,

Diameters.		Areas.	Content.
AB	48	6.417 × 3	19.251
LL	46	5.893 × 10	58.93
OO	30	2.506 × 10	25.06
CD	24		
		Cont.	103.24

Dep.	Content.
23	103.24
22	96.823
21	90.406
20	83.989
19	78.096
18	72.203
17	66.310
16	60.417
15	54.524
14	48.631
13	42.738
12	36.845
11	30.952
10	25.059
10	25.059
9	22.553
8	20.047
7	17.541
6	15.035
5	12.529
4	10.023
3	7.517
2	5.011
1	2.505
0	0.000

Fig. 66. In this latter table, the mean area at top 6.417, is ſubtracted continually from 103.24 for 3 terms. And then the mean area 5.893 is ſubtracted continually from 83.989 for 10 terms; and laſtly, the mean area 2.506 is ſubtracted continually from 25.059, for ten terms, and at laſt 0 remains.

Then to find how much liquor there is in the caſk at any time; you come to the caſk or tun, and take the depth of the liquor, ſuppoſe 15 inches; look for it on the ſide of the table, and againſt it is 54.524 gal. and ſo much liquor there is in the tun.

If the tun be very crooked, you muſt take a diameter in the middle of every 6 inches. And if there be a crown or fall in it, remember to make allowance for it.

2. *Otherwiſe thus.*

Make a table with the depth in inches on the ſide, as before, and the diameters againſt them in the ſecond column; which are found thus.

Divide the difference of the diameters in any fruſtum, by the number of inches in the fruſtum's height, commonly ten; and ſubtract the quotient continually from the greateſt diameter, or add it to the leaſt, as oft as there are terms in the fruſtum; and you will get all the diameters in the fruſtum for every inch. Do the ſame with every fruſtum. Then putting them down, in the ſecond column; place their areas againſt them in the third column; which areas are moſt eaſily got from a table of areas. Then add up theſe areas one by one, beginning at the loweſt, and place them in a 4th column, which will then ſhew the liquor in the caſk at any depth.

Diame-

Fig. 66.

Diameters.		differences.	ad. num.
AB	48	2 ÷ 8	0.25
LL	46	16 ÷ 10	1.6
OO	30	6 ÷ 5	1.2
CD	24		

Dep.	Diameters.	Areas.	Content.
23	48.	6.417	103.249
22	47.75	6.350	96.832
21	47.5	6.284	90.482
20	47.25	6.218	84.198
19	47.	6.152	77.980
18	46.75	6.087	71.828
17	46.5	6.022	65.741
16	46.25	5.955	59.719
15	46.	5.893	53.764
14	44.4	5.490	47.871
13	42.8	5.102	42.381
12	41.2	4.727	37.279
11	39.6	4.368	32.552
10	38.	4.622	28.184
9	36.4	3.690	23.562
8	34.8	3.373	19.872
7	33.2	3.070	16.499
6	31.6	2.781	13.429
5	30.	2.506	10.648
4	28.8	2.310	8.142
3	27.6	2.122	5.832
2	26.4	1.941	3.710
1	25.2	1.769	1.769
0	24	0.	

Fig. 66. Here 1.2 is continually added to 24, till we come at 30; then 1.6 is continually added to 30, and ſo on till 46. And then .25 is continually added from 46 to 48. Then the next column contains the areas, correſpondent to theſe diameters. And the laſt contains the ſum of all.

This way is more exact than the former, but more troubleſome.

The diameters required, are had by meaſuring them; but if the form of the veſſel is known, they may be had more exactly by calculation.

Cor. *When the veſſel is very broad, as in a cooler; it is ſometimes neceſſary, to meaſure it to every tenth part of an inch.*

This will be done the ſame way as the inching was done, only inſtead of the number of inches in the height, we muſt take the number of tenths, which will be 10 times greater than before, or conſiſt of 10 times more terms. And the quantity to be added or ſubtracted, for finding each diameter or area, will be only the tenth part of what it is for inching: but all the reſt of the proceſs will be the ſame; for the area muſt be found at the middle of every inch in height; and then add (or ſubtract) continually the tenth part of every area, to the preceding.

PROB.

Fig. 67.

PROB. XVIII.

To gauge a caſk ABHG.

1. RULE.

To $1\frac{1}{2}$ the ſum of the ſquares of the bung and head diameters CD, AB, add $\frac{5}{10}$, $\frac{3}{10}$ or $\frac{1}{10}$, or ſubtract $\frac{1}{10}$ their difference (according as the ſtaves are more or leſs crooked); multiply this ſum by $\frac{1}{3}$ the length EF, which divide by the proper diviſor, 359 for ale, or 294 for wine, gives the content in gallons.

2. *Or thus.*

To the head diameter AB, add $\frac{5}{10}$, $\frac{6}{10}$ or $\frac{7}{10}$, of the difference of the bung and head diameters, CD, AB (according as the ſtaves are more ſtreight or more crooked); the ſum is the mean diameter. Multiply the ſquare of the mean diameter by the length of the caſk EF, and divide the product by 359 or 294, gives the ale or wine gallons reſpectively, contained in it.

3. *Or thus.*

When the form of the caſk is known, which ſeldom happens; compute its content in inches, by its proper rule in the 4th part before going; and dividing by the proper diviſor (Prob. I.), you'll have the gallons it contains. Or proceed by Prob. 26, Part IV, by taking a diameter in the middle between AB and CD.

Fig. 67.

Ex. 1.

There is a caſk whoſe bung diameter is 32 *inches, the head* 24, *and length* 40; *the ſtave pretty ſtreight, what is the content in ale gallons?*

	32	24	1024	3)40($13\frac{1}{3}$
	32	24	576	
	64	96	2)1600	
	96	48	800	
	1024	576	2400	
			—44.8 = $\frac{1}{10}$ dif.	
	1024		2355.2	
diff.	576		$13\frac{1}{3} = \frac{40}{3}$.	
	448		7065.6	
			23552	
			785.1	
			359)31402.7(87.4 ale gall.	

Fig. 67.

Examp. 2.

There is a caſk, whoſe bung diameter is 40 *inches, the head* 30, *and length* 54; *the ſtaves crooked. How much wine does it hold?*

$$\begin{array}{lrrl}
40\text{ ſquare} = & 1600 & 2500 & \\
30\text{ ſquare} = & 900 & 1250 & \\ \cline{2-3}
\text{ſum} & 2500 & 3750 & \\ \cline{2-2}
 & & 210 & \\ \cline{3-3}
\text{diff.} & 700 & & \\
 & 3 & 3960 & \\ \cline{2-2}
 & 210.0 & 18 & = \frac{54}{3} \\ \cline{2-2}
 & & 31680 & \\
 & & 3960 & \\ \cline{3-3}
\end{array}$$

294)71280(241.7 wine gall.

If the ſtaves were very crooked, you muſt have added $\frac{5}{10}$ of 700, or 350; and then the content would be 250.2 gall.

Or thus.

$$\begin{array}{rrl}
40 & 30 & \\
30 & 6 & \\ \cline{1-2}
10 & 36 & = \text{m. diam.} \\
.6 & 36 & \\ \cline{1-2}
6.0 & 216 & \\ \cline{1-1}
 & 108 & \\ \cline{2-2}
 & 1296 & \\ \cline{2-2}
\end{array}
\qquad
\begin{array}{r}
1296 \\
54 \\ \hline
5184 \\
6480 \\ \hline
\end{array}$$

294)69984(238. wine gall.

By

Fig. 67.

By the Rule.

Set 17.14 on D, to the height 54 on C; and againſt the mean diameter 36 on D, is the content 238 on C.

Note, the mean diameter may be found on the rule, according to its variety.

This alſo may be done by the diagonal line on the gauge rod, for putting the rod in at the bung hole, to the oppoſite corner, at either end; note where the middle of the bung hole cuts the line, and there you have the number of gallons which the caſk contains.

Or thus, having the diagonal in inches; ſet 22.93 upon D, to 27 upon E, for ale; or ſet 15.9 upon D, to 11 upon E, for wine; and againſt the diagonal upon D, you have the content upon E.

SCHOLIUM.

It has been the common practice of Gaugers, tō reduce all caſks to the conic ſections, ſuch as the ſpheroid, the paraboloid, the hyperboloid, the parabolic ſpindle, &c. and upon that ſuppoſition, to make ſo many caſes or varieties, for finding the contents of all ſorts of caſks, by reducing them to one or other of theſe. But it is impoſſible, by the mere ſight of a cask, to know what form to refer it to, and it muſt be mere chance if it happen to be any of them; therefore it is better to have regard only to the curvature of the ſtaves, ſince it is owing entirely to this, that a cask holds more or leſs; and therefore from this only a judgment may be formed how to proceed by theſe rules, in finding a mean area, or a mean diameter, for reducing a cask to a cylinder; or without truſting to the eye, to take a middle diameter (according to Prob. 26. Part 4.) which will determine the thing at once.

In order to get the dimenſions of any cloſe caſk; Fig. put a rod in at the bung hole, ſtreight to the op- 67. poſite ſide, and note the length of the rod on the outſide of the caſk. From this, take the thickneſs of a ſtaff (which is eaſily taken with a notch'd ſtick) and the remainder is the bung diameter.

The head diameter is had, by taking twice the thickneſs of a ſtaff (found at the bung hole) from the external diameter at the head. Or the diſtance between the inſide of the chine, and the outſide of the oppoſite ſtaff, will be nearly equal to the head diameter.

Take the diſtance between the outſides of the two heads, with a ſliding pair of calipers; from that ſubtract twice the thickneſs of a head, the remainder is the length of the caſk within.

The outſide diameters are eaſily found, by meaſuring round the caſk to get the circumferences; then from any circumference find the diameter.

To find the wet inches in a caſk, put the gauging rod into the bung hole, in a lying caſk; or into the end, in a ſtanding one; and thruſt it directly down, till it touch the oppoſite ſide or end; then taking it out, the wet part will ſhew how deep the liquor is, or the wet inches.

PROB. XIX.

To find the ullage of a caſk, whoſe axis lies parallel to the horizon.

The *ullage* is the quantity of liquor remaining 68. in the caſk; or the vacuity, when the reſt is drawn out.

RULE.

Here muſt be given the bung diameter, the wet or dry inches, and the content of the caſk.

Fig. 68. Divide the wet inches by the bung diameter, and note the quotient: take a quarter of the difference between this quotient and $\frac{5}{10}$, and add it to that quotient, if greater than .5, or ſubtract it if leſſer. By this, multiply the whole content of the cask, and it gives the liquor remaining in it.

But when the bung diameter is far greater than the head, inſtead of $\frac{1}{4}$, take $\frac{1}{3}$ the difference aforeſaid.

Examp.

Suppoſe the bung diameter CD 32, *the wet part* LD 24. *Then by Prob.* 18, *the content of the caſk is* 97.46. *To find the liquor contained in* IKHB, *or the ullage.*

```
32)24.00(.75        .75            97.46
   224              .5              .812
   ———              ———            —————
    160         4).25(.062         77968
    160              .75             975
    ———             ————             195
                    .812           —————
                            ullage 79.138
                                   —————
```

By the Rule.

Set the bung diameter 32 upon C, to 100 upon SL; and againſt the wet inches 24 on C, is a ſegment 82.25 on SL.

Then ſet 100 on A, to the cask's content 97.46 on B; and againſt the ſegment on A, is the ullage 80 on B.

PROB. XX.

To find the ullage of a caſk, whoſe axis is perpendicular to the horizon.

1. RULE.

69. Here we muſt have the length of the caſk, the wet or dry inches, and the content of the caſk.

Divide the wet inches by the cask's length, and Fig.
note the quotient; take $\frac{1}{10}$ the difference between 69.
this quotient and $\frac{5}{10}$, and add it to that quotient, if greater than .5, or ſubtract it if leſſer; by this, multiply the whole content of the cask, and you have the liquor remaining in the cask.

2. *Or thus.*

Here you muſt have the diameters at the bung, head, and liquor's ſurface; alſo the wet inches, and whole length.

Then by Prob. 18. find the content of IKGH, by finding a mean diameter; and this is the ullage.

Or find the content of the half cask CDHG, and of the part CDKI; then the ſum or difference gives the quantity of liquor in the cask.

Or laſtly, find a middle diameter between IK and GH, and then find the content by Rule 1. Prob. 16.

Examp.

Suppoſe the bung diameter CD 32, *the head* GH 26, *the length* EF 40, *the wet inches* LF 12, *ſurface of the liquor* IK 31.5; *to find the quantity of liquor in* IGHK.

By Prob. 18. the content of the cask is 100.6 ale gallons.

```
40)12.0(.3      .5        .30          100.6
                .3        .02           .28
                ——        ———          —————
          10).2(.02       .28          8048
                          ———          2012
                                      ——————
                               ullage 28.168
                                      ——————
```

Or thus.

Fig. 69.

IK = 31.5
GH = 26.

diff. 5.5
.5

2.75
26.

m. diam. 28.750
28.75

57500
23000
2012
144

826.56

826.56
12

165312
82656

359)9918.72(27.6 ale gallons.

By the Rule.

Set the cask's length 40 on C, to 100 on SS; and againſt the wet inches 12 on C, is a ſegment 26.4 on SS.

Set 100 on A, to the cask's content 100.6 on B; and againſt that ſegment 26.4 on A, is the ullage 28 on B.

PROB. XXI.

To gauge malt in any maſh tun, ciſtern or couch; or on any floor.

RULE.

In a *Tun*, *Ciſtern*, or *Couch*, if the malt does not lie level; take a ſufficient number of depths in different places; and divide the ſum of them by their number, and it will give the mean depth.

Then find the area at top or ſurface of the malt, and at the bottom; and take half the ſum, for the mean area; and multiply the mean area by the mean depth, and divide by 2150.4, gives the content in buſhels. See Prob. 16. Fig. 69.

Upon a *floor*, find the area of the baſe, and the mean depth (by ſeveral dippings); and multiply them together, and divide by 2150.4, as before. This work is the ſame as directed in Prob. 10. Part IV.

When the dimenſions are large, they are commonly taken with a tape divided into inches.

It has been obſerved, that when 2 or 3 worts have paſſed thro' malt of a middling ſort, it will be more compact by a ſixth or an eighth part. And therefore the quantity found in ſuch circumſtances, ſhould be increaſed ſo much.

Examp.

Suppoſe the length of a ciſtern 85, *and breadth* 54.4, *and depth* 18; *what does it contain?*

54.4
85
———
2720
4352
———
4624.0
18
———
36992
4624
———
2150.4(83232(38.7 content.

1. *By the Rule.*

Find a mean between 54.4 and 85, which will be 68; then ſet 46.37 on D, to the depth 18 on C; and againſt the mean 68 on D, is the content 38.7 on C.

2. *Or*

Fig. 69.

2. *Or thus, by the Rule.*

Set the depth 18 on MD, to the length 85 on B; and againſt the breadth 54.4 on A, is the content 38.7 on B.

SCHOL.

It may be obſerved, that many of the rules laid down here for gauging, are only approximations; being contrived for eaſe and expedition; and come pretty near the truth in moſt caſes. For to what purpoſe can it be to make uſe of long and laborious rules and calculations, ſuited to certain particular figures; when we do not know that any veſſels we deal with have theſe figures. If we ſhould gauge a cask as a paraboloid, which happens to be a parabolic ſpindle; we ſhould make a great miſtake. Therefore it is better in mechanical things to uſe mechanical rules; and judge from the apparent curvature of the veſſel, what rule to uſe, and how to apply it.

FINIS.

Fig. 69.

2. *Or thus, by the Rule.*

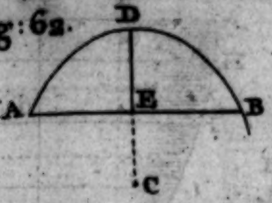
Set the depth 18 on MD, to the length 85 on B; and againſt the breadth 54.4 on A, is the content 38.7 on B.

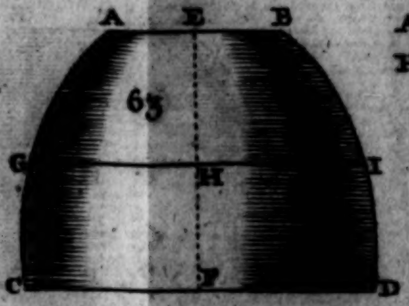
SCHOL.

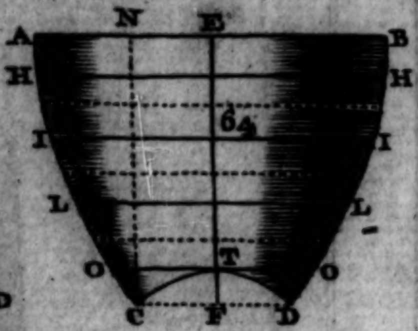
It may be obſerved, that many of the rules laid down here for gauging, are only approximations; being contrived for eaſe and expedition; and come pretty near the truth in moſt caſes. For to what purpoſe can it be to make uſe of long and laborious rules and calculations, ſuited to certain particular figures; when we do not know that any veſſels we deal with have theſe figures. If we ſhould gauge a caſk as a paraboloid, which happens to be a parabolic ſpindle; we ſhould make a great miſtake. Therefore it is better in mechanical things to uſe mechanical rules; and judge from the apparent curvature of the veſſel, what rule to uſe, and how to apply it.

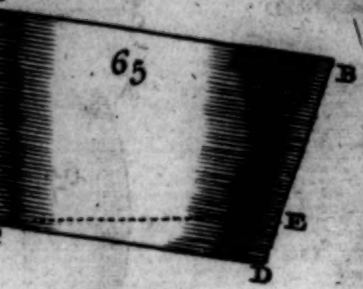
FINIS.

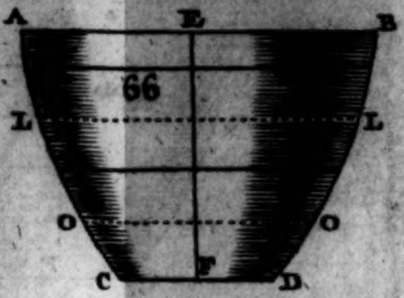
ig: 62.

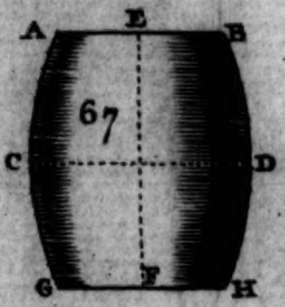


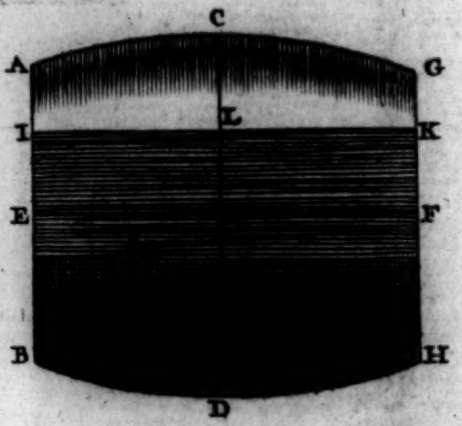


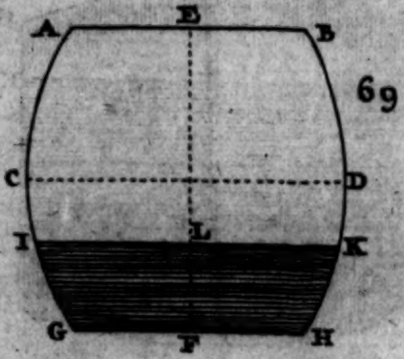












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